concrete INFORMATION

CIRCULAR CONCRETE TANKS WITHOUT PRESTRESSING

Section 1. Introduction

Design data for circular tanks built in or on ground have been confined almost entirely to walls with triangular load distribution, the top being unrestrained and the base assumed fixed. The assumption of fixed base is not generally in conformity with the actual conditions of restraint, and for other conditions there are little or no published data for designers to use. Adjustments then had to be made in accordance with the judgment of the engineer.

Existing data have been simplified and greatly augmented in this publication so that cylindrical concrete tank walls can now be designed for bases that are fixed, hinged, or have any other degree of restraint. A procedure is illustrated by which restraining moments at the edges can be determined and stresses due to such moments can be computed. The effect of radial displacements at the edges is also treated, and design data are given for trapezoidal distribution of loading on the walls.

Various layouts for circular roof slabs supported on tank walls are discussed and their design illustrated. Detailed designs are given for slabs without interior support as well as for slabs with one, four or seven columns. Various degrees of restraint at the edge of the slabs are included in the designs.

The subject is divided into sections, each of which deals with just one major phase of the design. The discussion in Sections 4 through 15 is given in connection with numerical examples most of which apply to a tank having the same dimensions. In order to design a particular tank, it is of course not necessary to make all the calculations in all the 12 sections. For example, to design an open-top tank supported on an ordinary wall footing, follow Section 5 and apply the adjustment for radial displacement discussed in Section 8. If the tank wall carries a roof slab, say, without interior supports, add the calculations illustrated in Section 11 for the slab and, if the slab and wall are continuous, also make calculations as in Section 9.

In general, by proper combination of various sections, one can design numerous tanks involving many sets of conditions which cover practically the entire field of construction of cylindrical tanks built in or on ground. A typical example is given in detail in Section 16.

Section 2. Proportioning of Sections Subject to Ring Tension and Shrinkage

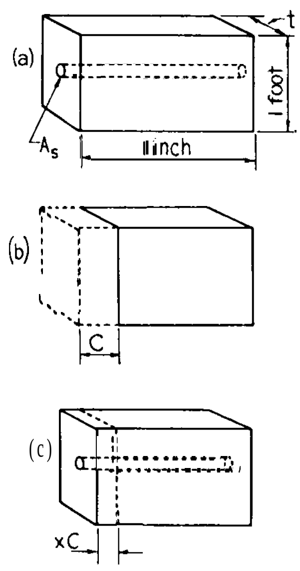

FIG. 1

Formulas for stresses in a reinforced concrete ring due to shrinkage will be derived first. Fig. 1 (a) illustrates a block of concrete reinforced with a bar as shown but otherwise unrestrained. The height of the block is chosen as 1 ft. since tension in a circular ring of a tank wall is computed for that height. The dimension marked t corresponds to the wall thickness. The steel area is A_s and the steel percentage is p.

If the bar is left out as in Fig. 1 (b), shrinkage will shorten the l-in. long block a distance of C, which denotes the shrinkage per

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unit. The presence of the steel bar prevents some of the shortening of the concrete, so the difference in length of the block in Fig. 1 (b) and Fig. 1 (c) is a distance xC, in which x is an unknown quantity.

Compared with (b), the concrete in (c) is elongated a distance xC from its unstressed condition, so the concrete stress is

$$f_{cs} = xCE_c$$

Compared with (a), the steel in (c) is shortened a distance (1 − x) C from its unstressed condition, so the steel stress is

$$f_{ss} = (1 - x)\, CE_s$$

The total tension in the concrete equals the total compression in the steel, so $pf_{ss} = f_{cs}$. The stresses derived from these equations are

$$f_{ss} = CE_s \frac{1}{1 + np} \text{ (compression)}$$

$$f_{cs} = CE_s \frac{p}{1 + np} \text{ (tension)}$$

The concrete stress due to a ring tension, T, is practically equal to T/A_c $(1 + np)$, and the combined concrete tensile stress equals

$$f_c = \frac{CE_sA_s + T}{A_c + nA_s} \quad (1)$$

This formula will be used repeatedly to investigate ring stresses in circular walls.

The usual procedure in tank design is to provide horizontal steel, A_s, for all the ring tension at a certain allowable stress, f_s, as though designing for a cracked section. After determining $A_s = T/f_s$, the concrete tensile stress in the uncracked section due to combined ring tension and shrinkage is checked by inserting the value of A_s in Equation 1. By setting A_c = 12t (t in in.), and solving for t:

$$t = \frac{CE_s + f_s - nf_c}{12 f_c f_s} \times T \quad (2)$$

This formula may be used to estimate the wall thickness. For illustration assuming the shrinkage coefficient, C, of concrete as 0.0003:

$$t = \frac{0.0003 \times 30 \times 10^6 + 14{,}000 - 10 \times 300}{12 \times 300 \times 14{,}000} \times T$$

$$= \frac{9{,}000 + 14{,}000 - 3{,}000}{50{,}400{,}000} \times T = 0.0004T$$

It is felt that an allowable concrete tensile stress for cylindrical tank design of 300 p.s.i. for a 3,000-lb. concrete is a reasonably conservative value when shrinkage is included and ring tension is determined on basis of a carefully conducted analysis.

Section 3. Allowable Steel Stress in Ring Tension

Two objectives stand out in the design of cylindrical tank walls. The wall thickness should be sufficient to keep the concrete from cracking, but if the concrete does crack, the ring steel must be able to carry all the ring tension alone.

Allowable steel stress for ring tension is often kept as low as 12,000 or even 10,000 p.s.i. It will be demonstrated that lowering the allowable steel stress actually tends to make the concrete crack because the lower the steel stress the greater the area of steel provided and hence the higher the concrete stress due to shrinkage. Inserting $A_s = T/f_s$ in Equation 1 gives

$$f_c = \frac{CE_s + f_s}{A_c f_s + nT} \times T$$

For illustration, use the data given in Section 4, assume T = 24,100 lb., and compute f_c for values of f_s as given below.

f_s	10,000	12,000	14,000	16,000	18,000	20,000	Infinity*
f_c	318	301	266	279	271	264	201

*When f_s = infinity, $A_s = 0$ and $f_c = T/A_c$.

NOMENCLATURE

- A: area (sq.in.)-used with various subscripts
- a: coefficient-used in $A_s = M/ad$
- b: width (in.)
- C: shrinkage coefficient
- c: diameter of drop panel
- D: diameter of tank (ft.)
- d: effective depth (in.)
- E: modulus of elasticity (p.s.i.)—used with various subscripts
- e: coefficient of thermal expansion
- F: equals $bd^2/12{,}000$
- f: stress (p.s.i.)—used with various subscripts
- H: height of wall (ft.)
- k: stiffness factor-also thermal conductivity coefficient-used with various subscripts
- L: span length (ft.)
- M: moment (ft.lb. or ft.kips)-used with various subscripts
- n: E_s/E_c
- p: load or pressure (lb. per sq.ft.)-also percentage of steel
- R: radius of tank (ft.)
- s: surface thermal coefficient
- T: ring tension (lb.)-also temperature-used with various subscripts
- t: thickness of tank wall (ft. or in.)
- V: total shear (lb.)
- v: unit shear (p.s.i.)
- W: total load on a panel (lb.)
- w: weight of liquid (lb. per cu.ft.)

If the allowable steel stress is reduced from 20,000 to 10,000 p.s.i., the concrete tensile stress is actually increased from 264 to 318 p.s.i., which is more than a 20 per cent increase. The lower the allowable steel stress, the sooner the concrete will crack. From this point of view, it is desirable to use higher allowable steel stresses.

It has been claimed that low steel stresses are necessary to minimize the width of cracks that may develop. It is of course true that a lower stress gives a smaller crack, but it is unwise to focus attention on this point exclusively. Other and probably more important points should be considered. Refer for illustration to the two sketches (a) and (b) in Fig. 2. Assume

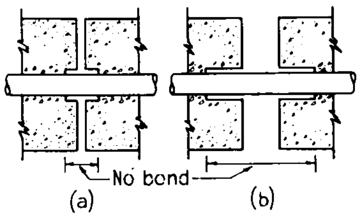


FIG. 2

that the bond resistance is higher in (a) than in (b), so that the bond may be considered broken as indicated. If the ratio of the lengths marked "no bond" is, say, 1:4, the ratio of the crack widths will also be 1:4. It is clear that the bond quality is of major importance in regard to width of cracks and leakage through cracks. The unit stress in the steel is a factor to be considered, but it seems to be subordinate to the question of bond.

In conventional tank construction, one of the first objectives should be to provide the best possible bond between concrete and reinforcement. The bars should, of course, be deformed and of the smallest size possible. The importance of this recommendation will be illustrated. One square inch of cross-sectional area in various bar sizes has the following bond areas in sq.in. per in. length:

⅜φ	½φ	⅝φ	¾φ	⅞φ	1φ	1□	1⅛□	1¼□
10.70	7.86	6.33	5.36	4.58	4.00	4.00	3.54	3.20

It is seen that 1 sq.in. of steel provided in the ¾-in. size has nearly 20 per cent more bond area-and bond resistance-than 1 sq.in. of ⅞-in. bars. The apparent benefit in regard to reduction of crack width obtained by reducing the allowable steel stress from, say, 12,000 to 10,000 will be nullified if this makes it necessary to increase the bar size from ¾ to ⅞ in.

Section 4. Wall with Fixed Base and Free Top -Triangular load

In past practice, walls built continuous with their footings have generally been designed as though the base were fixed and the top free. Design curves were available only for these conditions combined with a triangular distribution of pressure. It will be shown later that the base is seldom fixed, but it is helpful to start with this assumption and then go on to design procedures for other more correct conditions.

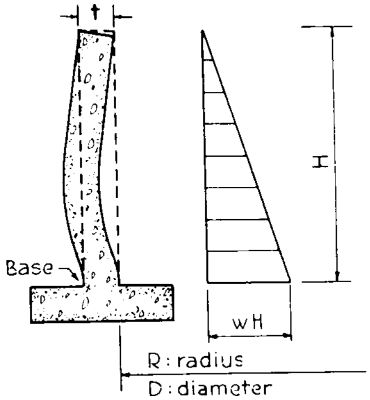


FIG. 3

The numerical values below are used in this and in subsequent sections that deal with various phases of tank wall design:

Height, $H = 20.0$ ft.
Diameter to inside of wall, D = 54.0 ft.
Weight of liquid, w = 62.5 lb. per cu.ft.
Shrinkage coefficient, C = 0.0003
Modulus of elasticity of steel, $E_s = 30 \times 10^6$ p.s.i.
Allowable concrete tensile stress for cylindrical tank design, $0.10f'_c$ = 300 p.s.i.
Ratio of moduli of elasticity, n = 10
Allowable stress in ring steel, f_s = 14,000 p.s.i.
Allowable stress in steel elsewhere, f_s = 20,000 p.s.i.

For the wall with fixed base in Fig. 3, estimate the wall thickness as t = 10 in. or 0.83 ft., and compute

$$\frac{H^2}{Dt} = \frac{20^2}{54.0 \times 0.83} = 8.9, \text{ say } 9^*$$

In this calculation, it would be more correct to take D to the center of the wall, that is, to use $D = 54.83$ instead of 54.0. This would give a value of 8.8, but the difference is so small that it will be disregarded.

Ring tension per ft. of height will be computed by multiplying wHR by the coefficients for $H^2/Dt = 9$ taken from Table I.

$wHR = 62.5 \times 20.0 \times 27.0 = 33{,}750$ lb. per ft.

This is the ring tension that would exist at the base if it could slide freely.

The coefficients and the ring tension are as follows, Point $0.0H$ denoting the top and Point $1.0H$ the base of the wall.

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Coef., Table I**	-0.011	+0.101	+0.213	+0.329	+0.440	+0.536	+0.591	+0.559	+0.410	+0.165	0
Ring ten.	-400	+3,400	+7,200	+11,100	+14,900	+18,200	+19,900	+18,900	+13,800	+5,600	0

*H^2/Dt is a common factor involved in all values of ring tension and moment and is therefore a convenient characteristic to use in entering the accompanying design tables.

**For convenience a straight-line interpolation has been used which gives sufficient accuracy.

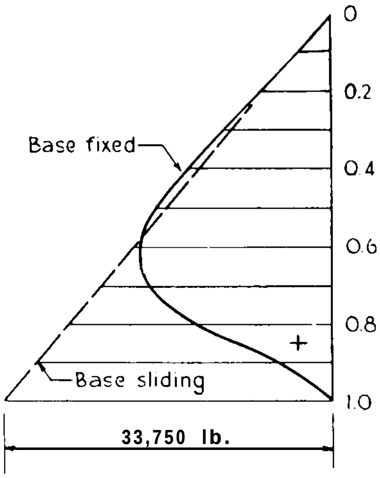


FIG. 4

A plus sign denotes tension, so there is compression at the top but it is negligible. The ring tension is zero at the base since it is assumed that the base has no radial displacement. Ring tension values at the tenth-points are plotted in Fig. 4.

ing the effect of shrinkage (see Section 2) is

$$f_c = \frac{CE_sA_s + T_{max.}}{A_c + nA_s} = \frac{0.0003 \times 30 \times 10^6 \times 1.42 + 19{,}900}{10 \times 12 + 10 \times 1.42}$$

$$= \frac{12{,}800 + 19{,}900}{134.2} = 244 \text{ p.s.i.}$$

Since 300 p.s.i. is considered allowable, the 10-in. wall thickness is ample. The fact that almost 40 per cent of the total tensile stress is due to shrinkage shows how important it is to reduce shrinkage in tank wall construction.

Shear at base of wall equals wH^2 multiplied by the coefficient of 0.166 for $H^2/Dt = 9$ taken from Table XVI.

$$V = 0.166 \times wH^2 = 0.166 \times 62.5 \times 20^2 = 4{,}150 \text{ lb. per ft.}$$

$$v = \frac{V}{0.875bd} = \frac{4{,}150}{0.875 \times 12 \times 8} = 49 \text{ p.s.i.}$$

Moments in vertical wall strips 1 ft. wide are computed by multiplying wH^3 by the coefficients taken from Table VII.

$$wH^3 = 62.5 \times 20^3 = 500{,}000 \text{ ft.lb. per ft.}$$

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Coef., Table VII	0	0	0	+—002	+—006	+—012	+0.0024	+0.0034	+0.0029	—017	-0.0134
Mom.	0	0	0	+0.00100	+0.00300	+0.00600	+ 1,200	+ 1,700	+ 1,500	+0.00900	- 6,700

It is seen that maximum ring tension, + 19,900 lb., occurs at Point $0.6H$. Above that point the difference between the curves marked "Base fixed" and "Base sliding" is relatively small. In that part of the wall it is evident that the condition of restraint at the base has but little effect. Below Point $0.6H$, however, there is a large difference between the two curves. With a sliding base, the maximum ring tension for which the wall is to be designed is 33,750 lb. This is 70 per cent more than the maximum ring tension for the fixed base.

The assumption of fixed base obviously gives an economical design but, as will be illustrated later, the assumption may often give a design that is unsafe. It is not recommended for application to tank wall design without proper investigation. Before using the fixed base assumption, ascertain first that both displacement and rotation are fully prevented at the base. Studies presented in subsequent sections serve to demonstrate that it is difficult to "fix" the base.

In accordance with Section 2, maximum area of ring steel is

$$A_s = \frac{T_{max.}}{f_s} = \frac{19{,}900}{14{,}000} = 1.42 \text{ sq.in. per ft.}$$

Use $\frac{5}{8}$-in. round bars spaced $5\frac{1}{4}$ in. o.c. in each of two curtains, giving a total area of 1.42 sq.in. Determine reinforcement elsewhere in the wall to fit the ring tension curve in Fig. 4.

Maximum tensile stress in the concrete includ-

Negative sign denotes tension in the inside. The moments are plotted in Fig. 5 from which it is seen that tension in the inside of the wall extends only to a point about 0.12 X 20 = 2.4 ft. above the base. Bars

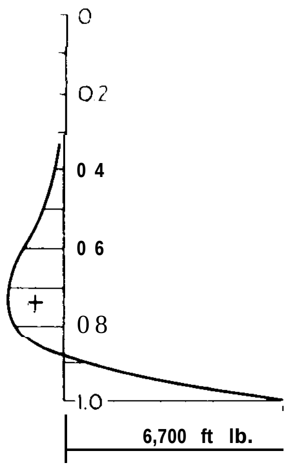


FIG. 5

in the inside need only extend either 33 diameters or 2.4 ft. + 12 diameters, whichever is the greater, above the base. Their area is

$$A_s = \frac{M^*}{ad} = \frac{6.7}{1.44 \times 8} = 0.58 \text{ sq.in.}$$

*$a = f_sj/12{,}000$, see *Reinforced Concrete Design Handbook* of the American Concrete Institute.

Use $\frac{7}{8}$-in. round bars spaced 12 in. o.c. and extend them 2.4 + 12 diameters = 3.3 ft. above the base. Above that, only nominal vertical reinforcement, say $\frac{1}{2}$-in. round bars spaced 18 in. o.c., is needed for support and spacing of the ring bars in the inside curtain. The area of vertical bars in the outside curtain is

$$A_s = \frac{M}{ad} = \frac{1.7}{1.44 \times 8} = 0.15 \text{ sq.in.}$$

Use $\frac{1}{2}$-in. round bars spaced 16 in. o.c. They serve both for tension bars and for support of ring bars during erection. This amount of outside vertical reinforcement is generally insufficient since the base is seldom fully fixed. It is better to determine these bars as in the example that follows.

Section 5. Wall with Hinged Base and Free Top-Triangular load

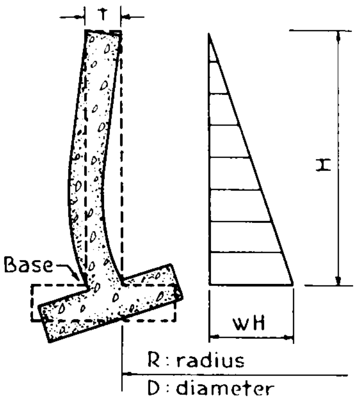


FIG. 6

The design in Section 4 is based on the assumption that the base joint is continuous and the footing is prevented from even the smallest rotation of the kind shown exaggerated in Fig. 6. The rotation required to reduce the fixed base moment from 6,700 ft.lb. per ft. to, say, zero is much smaller than rotations that may occur when normal settlement takes place in the subgrade. It is difficult to predict the behavior of the subgrade and its effect upon the restraint at the base, but it is more reasonable to assume that the base is hinged than fixed, and the hinged-base assumption gives a safer design.

Data for walls with hinged base are presented, and their application to the design of a tank wall with the same dimensions as in Section 4 is illustrated and the two designs compared.

Coefficients for ring tension taken from Table II (for $H^2/Dt = 9$) are multiplied by $wHR = 33{,}750$ lb. per ft., as in Section 4.

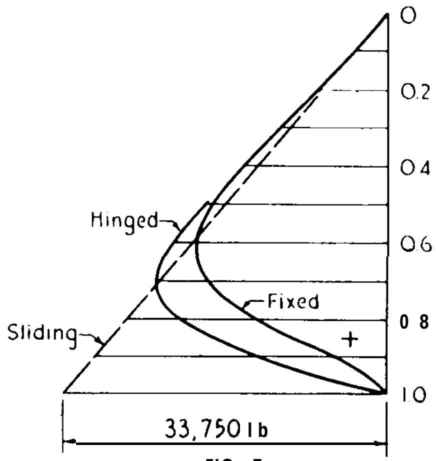


FIG. 7

In Fig. 7 comparison of ring tension is made for bases that are fixed, hinged or sliding. In the upper one-half of the wall the base condition has but little effect, but below Point $0.5H$ the difference between hinged and fixed base becomes increasingly larger. Maximum ring tension for hinged base, $T_{max.}$ = 24,100 lb., occurs at Point $0.7H$. It is 21 per cent larger than $T_{max.}$ for fixed base. A design based on assumption of "fixed base" and f_s = 12,000 p.s.i. will require practically the same steel area as a design made for "hinged base" and f_s = 14,500 p.s.i., but the latter procedure gives a distribution of steel area that corresponds more closely to the actual conditions. Tensile stress in the concrete will also be determined with greater accuracy.

Maximum area of ring steel is

$$A_s = \frac{T_{max.}}{f_s} = \frac{24{,}100}{14{,}000} = 1.72 \text{ sq.in. per ft.}$$

At Point $0.7H$, use $\frac{5}{8}$-in. round bars spaced $4\frac{1}{2}$ in. o.c. in each curtain of reinforcement (A_s = 1.66 sq.in.). Determine reinforcement'elsewhere to fit the ring tension curve.

The maximum tensile stress in the concrete including effect of shrinkage is:

$$f_c = \frac{CE_sA_s + T_{max.}}{A_c + nA_s} = \frac{0.0003 \times 30 \times 10^6 \times 1.66 + 24{,}100}{10 \times 12 + 10 \times 1.66}$$

$$= \frac{14{,}900 + 24{,}100}{136.6} = 285 \text{ p.s.i.}$$

Since 300 p.s.i. is considered allowable, the IO-in. wall thickness is sufficient.

For shear at base of wall, select coefficient from Table XVI :

$$V = 0.092 \times wH^2 = 0.092 \times 62.5 \times 20^2 = 2{,}300 \text{ lb. per ft.}$$

$$v = \frac{V}{0.875bd} = \frac{2{,}300}{0.875 \times 12 \times 8} = 27 \text{ p.s.i}$$

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.511	0.611	0.711	0.8H	0.9H	1.0H
Coef., Table II	−0.012	+0.096	+0.204	+ 0.318	+ 0.436	+ 0.558	+ 0.663	+ 0.713	+ 0.649	+ 0.409	0
Ring ten.	− 400	+3,200	+6,900	+10,700	+14,700	+18,800	+22,400	+24,100	+21,900	+13,800	0
Ring ten.. Sec. 4	− 400	+3,400	+7,200	+11,100	+14,900	+18,200	+19,900	+18,900	+13,800	+ 5,600	0

Coefficients for moments in vertical wall strips 1 ft. wide, taken from Table VIII are multiplied by wH^3 = 500,000 ft.lb. per ft.

Point	0.0H	0.1H	0.2H	0.3H	0.4~	0.5H	0.6H	0.711	0.8H	0.9H	1.0H
Coef., Table VIII	0	0	0	-0.0002	0	+0.0005	+0.0016	+0.0032	+0.0050	+0.0050	0
Mom.	0	0	0	- 100	0	+ 300	+ 800	+ 1,600	+ 2,500	+ 2,500	0
Mom., Sec. 4	0	0	0	+ 100	+300	+ 600	+ 1,200	+ 1,700	+ 1,500	- 900	- 6,700

Moments for both hinged and fixed base are plotted in Fig. 8. The difference is considerable in the lower part of the wall. The maximum moment for hinged base gives tension in the outside and equals 2,500 ft.lb. per ft. at Point 0.8H. The area of vertical steel in the outside curtain is

$$A_s = \frac{M}{ad} = \frac{2.50}{1.44 \times 8} = 0.22 \text{ sq.in.}$$

Use ½-in. round bars spaced 11 in. o.c. Alternate bars may stop at mid-height, but the other bars extend to top of the wall to serve as support for ring bars during erection.

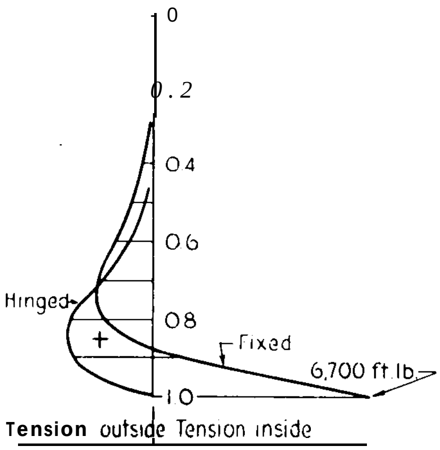


FIG. 8

The actual condition of restraint at a wall footing as in Figs. 3 and 6 is between fixed and hinged, but probably closer to hinged. The comparisons in Figs. 7 and 8 show that assuming the base hinged gives conservative although not wasteful design, and this assumption is therefore recommended. Nominal vertical reinforcement in the inside curtain lapped with short dowels across the base joints will suffice.

The procedure outlined in this section is considered satisfactory for open-top tanks with wall footings that are not continuous with the tank bottom, except that allowance should usually be made for a radial displacement of the footing. Such a displacement is discussed in Section 8. If the wall is made continuous at top or base, or at both, the continuity must also be considered. For these conditions refer to Sections 9 and 10.

Section 6. Wall with Hinged Base and Free Top-Trapezoidal load

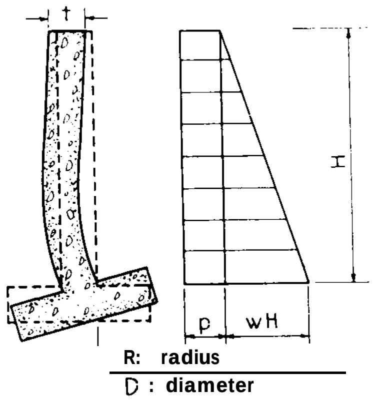


FIG. 9

In tanks used for storage of gasoline, check valves are often installed in order to reduce loss due to escape of gasoline vapor. The valve may be adjusted so that it takes a vapor pressure as high as 3 lb. per sq.in. to open it. Under such circumstances the pressure on the tank wall is a combination of the pressure due to the weight of the liquid plus a uniformly distributed loading due to the vapor pressure. The combined pressure on the wall has a trapezoidal distribution as shown in Fig. 9*, but it is convenient to separate it into two parts, a triangular element due to liquid weight and a rectangular element due to vapor pressure.

Design data for rectangular distribution of pressure may be useful also for design of tanks in which the liquid surface may rise considerably above the top of the wall, as may accidentally happen in tanks built underground.

In this section the design procedure for trapezoidal loading is illustrated. The data given in Section 4 are used, and in addition the vapor pressure is taken as p = 3 X 12^2 = 432 lb. per sq.ft. With this additional pressure, estimate t = 15 in., which is 5 in. more than in Sections 4 and 5. The investigation is made for hinged base.

For t = 15 in., or 1.25 ft. compute

$$\frac{H^2}{Dt} = \frac{20^2}{54 \times 1.25} = 5.9, \text{ say, } 6$$

Coefficients for ring tension are taken from

*Attention is called to the fact that the tank in Fig. 9 should have a roof when there is a surpressure on the liquid. Fig. 9 represents merely the loading conditions considered in this section; the effect of a roof slab is treated in subsequent sections.

Tables II and IV. They are multiplied by $wHR = 62.5 \times 20 \times 27 = 33{,}750$ lb. per ft. (triangular load, Table II) and by $pR = 432 \times 27 = 11{,}660$ lb. per ft. (rectangular load, Table IV).

$$= \frac{21{,}100 + 33{,}700}{203.4} = 269 \text{ p.s.i.}$$

Since 300 p.s.i. is considered allowable, the 15-in. thickness estimated is ample.

Point	0.0H	0.111	0.2H	0.3H	0.4H	0.511	0.611	0.7H	0.8H	0.911
Coef., Table II. Tri.	− 0.011	+ 0.103	+ 0.223	+ 0.343	+ 0.463	+ 0.566	+ 0.639	+ 0.643	+ 0.547	+ 0.327
Coef., Table IV. Rect.	+ 0.989	+ 1.003	+ 1.023	+ 1.043	+ 1.063	+ 1.066	+ 1.039	+ 0.943	+ 0.747	+ 0.427
Ring ten. Tri.	− 400	+ 3,500	+ 7,500	+11,600	+15,600	+19,100	+21,600	+21,700	+18,500	+11,000
Ring ten. Rect.	+11,500	+11,700	+11,900	+12,200	+12,400	+12,400	+12,100	+11,000	+ 8,700	+ 5,000
Total ring ten.	+11,100	+15,200	+19,400	+23,800	+28,000	+31,500	+33,700	+32,700	+27,200	+16,000

The total ring tension values are plotted in Fig. 10 together with the ring tension that would exist if the base joint could slide freely. The maximum tension for hinged base is 33,700 lb. per ft. (*see table above)* and occurs at Point $0.6H$. Above that point, it is seen that the change from hinged to sliding base does not have much effect. Below Point $0.6H$, ring tension for hinged base decreases rapidly until it becomes zero at the base. Actually, the condition at the base may be somewhere between hinged and freely sliding, so it is inadvisable to design the ring bars below Point $0.6H$

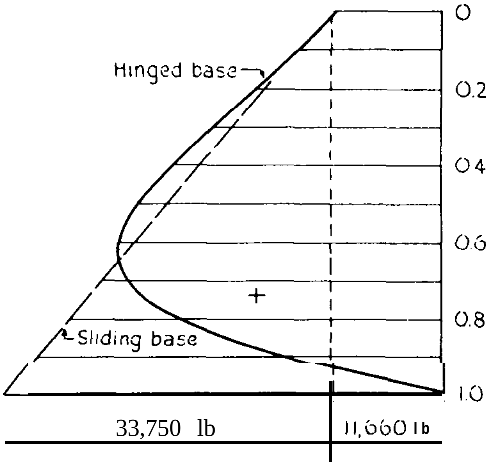


FIG. 10

for hinged base. The effect of a radial displacement at the base is discussed in Section 8.

Maximum area of ring steel is

$$A_s = \frac{T_{max.}}{f_s} = \frac{33{,}700}{14{,}000} = 2.41 \text{ sq.in.}$$

At Point $0.6H$, use ¾-in. round bars spaced 4½ in. o.c. in each of two curtains (A_s = 2.34 sq.in.).

Maximum tensile stress in the concrete including effect of shrinkage is

$$f_c = \frac{CE_sA_s + T_{max.}}{A_c + nA_s} = \frac{0.0003 \times 30 \times 10^6 \times 2.34 + 33{,}700}{15 \times 12 + 10 \times 2.34}$$

Shear at base of wall is determined on basis of coefficients taken from Table XVI for $H^2/Dt = 6$

$V = 0.110(wH^2 + pH) = 0.110\,(62.5 \times 20^2 + 432 \times 20)$
$= 3{,}700$ lb. per ft.

$$v = \frac{V}{0.875bd} = \frac{3{,}700}{0.875 \times 12 \times 13} = 27 \text{ p.s.i.}$$

Coefficients for moments per ft. of width, taken from Table VIII, are multiplied by

$$wH^3 + pH^2 = 62.5 \times 203 + 432 \times 20^2$$
$$= 673{,}000 \text{ ft.lb. per ft.}$$

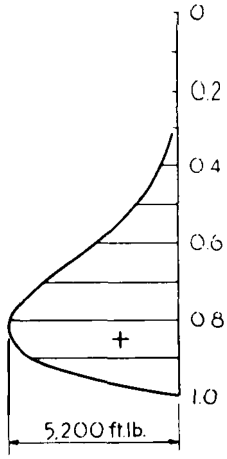
Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.711	0.8H	0.911	1.0H
Coef., Table VIII	0	0	0	+0.0002	+0.0008	+0.0019	+0.0039	+0.0062	+0.0078	+0.0068	0
Mom.	0	0	0	+ 100	+ 500	+ 1,300	+ 2,600	+ 4,200	+ 5,200	+ 4,600	0

These moments are plotted in Fig. 11. Maximum moment is 5,200 ft.lb. and occurs at Point $0.8H$. The maximum area of vertical steel in the outside curtain is

$$A_s = \frac{M}{ad} = \frac{5.20}{1.44 \times 13} = 0.28 \text{ sq.in. per ft.}$$

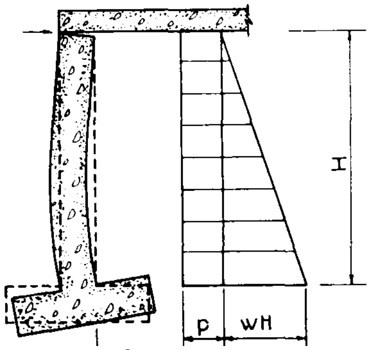
Use ½-in. round bars spaced 8½ in. o.c. (A_s = 0.28). All these bars shall extend to the bottom, but alternate bars may be discontinued near mid-height of the wall as indicated by the shape of the moment curve in Fig. 11.

FIG. 11



Nominal vertical reinforcement in the inside curtain lapped with short dowels across the base joint will suffice.

Section 7. Wall with Shear Applied at Top



R : radius
D : diameter

FIG. 12

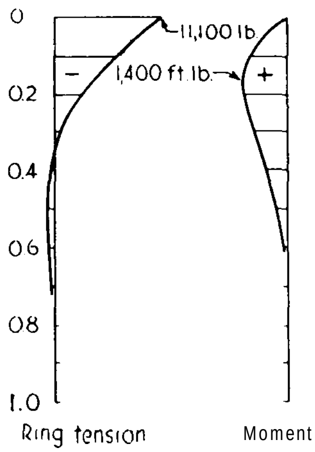


FIG. 13

As indicated in Fig. 12, the top of the wall may be doweled to the roof slab so that it cannot move freely as assumed in Sections 4, 5 and 6.

When displacement is prevented, the top cannot expand and the ring tension is zero at Point $0.0H$. In Section 6, with the top free to expand, the ring tension is 11,100 lb. at Point $0.0H$. To prevent displacement, add a shear at the top sufficient to eliminate the ring tension of 11,100 lb.

Ring tension due to a shear, V, at the top is computed by using coefficients in Table V for H^2/Dt = 6 and equals $-9.02VR/H$ lb. per ft. at the top. Therefore V must satisfy the equation

$$-9.02\frac{VR}{H} = -11{,}100$$

from which

$$V = \frac{11{,}100 \times H}{9.02R} = \frac{11{,}100 \times 20}{9.02 \times 27} = 910 \text{ lb. per ft.}$$

Table V is based on the assumption that the base is fixed. However, it will be seen later that the coefficient for H^2/Dt = 6 in Table V may be used with satisfactory accuracy also when the base is hinged.

For ring tension multiply coefficients in Table V by $VR/H = 910 \times 27/20 = 1{,}230$ lb. per ft., and for moment multiply coefficients in Table X by $VH = 910 \times 20 = 18{,}200$ ft.lb. per ft.

so small that they can be ignored. The effect of applying the shear at the top, therefore, is practically the same whether the base is fixed or hinged.

Ring tension and moment computed in this section are added to those in Section 6 and the results are plotted in Fig. 14. It is clear that the assumption of the top being free would be satisfactory in this case. It gives a conservative design for ring tension and

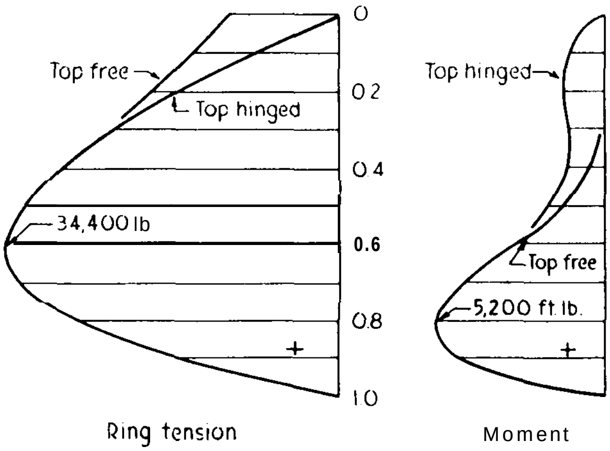


FIG. 14

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Coef., Table V	− 9.02	− 5.17	− 2.27	− 0.50	+ 0.34	+ 0.59	+ 0.53	+ 0.35	+ 0.17	+ 0.01	0
Ring ten.	−11,100	−6,400	−2,800	− 600	+ 400	+ 700	+ 700	+ 400	+ 200	0	0
Coef., Table X	0	+0.062	+0.070	+0.056	+0.036	+0.018	+0.006	0	−0.003	−0.005	−0.006
Mom.	0	+1,100	+1,300	+1,000	+ 700	+ 300	+ 100	0	− 100	− 100	− 100

Ring tension and moment are plotted in Fig. 13, the scale being the same as in Figs. 10 and 11. By comparing the ring tension curve in Fig. 13 with Fig. 10 and the moment curve with Fig. 11, it is seen that the values in the lower one-half of the wall in Fig. 13 are hardly affects the design for moments. Consequently, the investigation made in this section may be omitted in most cases with exception of tanks in which the ring tension is relatively large at the top and the wall is doweled to the roof slab.

Section 8. Wall with Shear Applied at Base

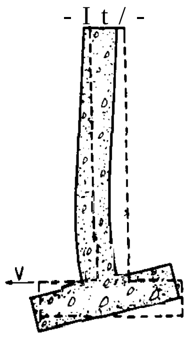


FIG. 15

Fig. 15 illustrates a case in which the base of the wall is displaced radially by application of a horizontal shear, V, which has an outward direction. When the base is hinged, the displacement is zero and the reaction on the wall is 3,700 lb. per ft. (see Section 6), which has an inward direction. When the base is sliding, the displacement is the largest possible, but the reaction is zero. For all intermediate displacements, the reaction must be between 0 and 3,700 lb. per ft.

It is difficult to ascertainwhether the footing is capable of providing a 3,700-lb. reaction without moving horizontally, but the chances are that it cannot do so in most instances. Any figure adopted for the displacement can be nothing more than a reasonable estimate. Since the extreme values of displacement occur when the reaction equals 0 (sliding base) and 3,700 lb. per ft. (hinged base), a reaction of 1,700 lb. has been chosen in this example as a reasonable value. This reaction may be obtained by superimposing two design conditions, one for hinged base, and the other for a shear of 3,700 − 1,700 = 2,000 lb. applied outwardly at the base. The procedure of design for shear at base will be demonstrated for this value.

The data in Table V are used for this investigation. It is true that the data are for a wall with shear applied *at one* end (top or base) while the other end (base or top) is fixed. It has been demonstrated for H^2/Dt = 6 in. in Section 7 that the conditions of restraint at one end have but little effect when applying a shear at the other end, so the data in Table V should give a good approximation also when the shear is applied at the base of the wall in Fig. 15.

For ring tension, multiply coefficients from Table V by $VR/H = 2{,}000 \times 27/20$ = 2,700 lb. per ft., and for moments multiply coefficients from Table X by $VH = 2{,}000 \times 20$ = 40,000 ft.lb. per ft. Coefficients in both tables are selected for the same value of H^2/Dt = 6 as in Sections 6 and 7.

As seen from Fig 16, it makes considerable difference whether the shear at the base is 3,700 or 3,700 − 2,000 = 1,700 lb. per ft. This cannot be ignored, but it is often possible to omit the investigation made in this section and still obtain a satisfactory solution. It is proposed to use the regular ring tension curve for hinged base only from the top down to *the* point of maximum tension, 33,700 lb. at $0.6H$, and to design all of the wall below that point for 33,700-lb. ring tension. This allowance is only slightly too high compared with the curve marked "Base displaced". The excess amount of ring tension is shown cross-hatched in Fig. 16. The difference between the moment curves appears to be considerable, but the moments are of relatively small importance, and the larger values for hinged base are preferred.

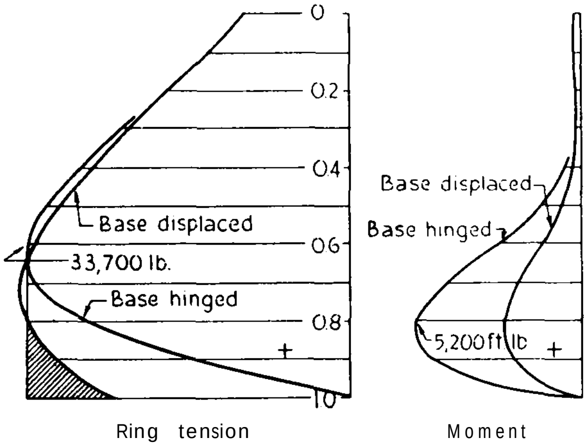


FIG. 16

The radial displacement corresponding to the 2,000-lb. shear *may* be determined as follows. The shear creates a ring tension of 24,300 at the base, and the unit stress on a transformed section of 203.4 sq.in. (see Section 6) equals $f_c = 24{,}300/203.4 = 119$ p.s.i. The corresponding unit strain equals $f_c/E_c = 119/3{,}000{,}000$ = 0.00004, so the radius has received an elongation of $R \times f_c/E_c = 27 \times 12 \times 0.00004 = 0.013$ in. In other words, the shear of $V = 2{,}000$ lb. per ft. causes the base to move horizontally a distance of only 1/77 in. It is clear that ordinary soil cannot offer much resistance against such a relatively small displacement. The major part of the resistance must be furnished by the concrete and circumferential reinforcement in the footing itself.

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Coef., Table V	0	− 0.01	− 0.17	− 0.35	− 0.53	− 0.59	− 0.34	+ 0.50	+ 2.27	+ 5.17	+ 9.02
Ring ten. V = 2000	0	0	− 500	− 900	− 1,400	− 1,600	− 900	+ 1,400	+ 6,100	+14,000	+24,300
Ring ten., Sec. 6	+11,100	+15,200	+19,400	+23,800	+28,000	+31,500	+33,700	+32,700	+27,200	+16,000	0
Total ring ten.	+11,100	+15,200	+18,900	+22,900	+26,600	+29,900	+32,800	+34,100	+33,300	+30,000	+24,300
Coef., Table X	+ 0.006	+ 0.005	+ 0.003	0	− 0.006	− 0.018	− 0.036	− 0.056	− 0.070	− 0.062	0
Mom. V = 2000	+ 200	+ 200	+ 100	0	− 200	− 700	− 1,400	− 2,200	− 2,800	− 2,500	0
Mom., Sec. 6	0	0	0	+ 100	+ 500	+ 1,300	+ 2,600	+ 4,200	+ 5,200	+ 4,600	0
Total mom.	+ 200	+ 200	+ 100	+ 100	+ 300	+ 600	+ 1,200	+ 2,000	+ 2,400	+ 2,100	0

Section 9. Wall with Moment Applied at Top

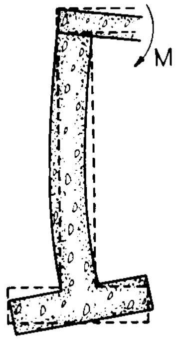


FIG. 17

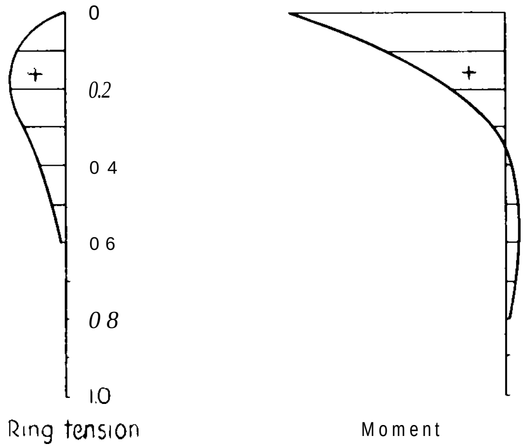


FIG. 18

When the top of the wall and the roof slab are made continuous, as indicated in Fig. 17, the deflection of the roof slab tends to rotate the top joint and introduces a moment at the top of the wall. In this section, the wall is investigated for a moment of $M = 6{,}700$ ft.lb. per ft., the origin of which is discussed later in this section.

The data in Tables VI and XI will be used although they are prepared for moment applied at one end of the wall when the other is free. However, these tables may be used with good degree of accuracy also when the far end is hinged or fixed. For ring tension, multiply coefficients from Table VI by $MR/H^2 = 6{,}700 \times 27/20^2 = 450$ lb. per ft., and for moments, multiply coefficients from Table XI by $M = 6{,}700$ ft.lb. per ft. Select coefficients for $H^2/Dt = 6$.

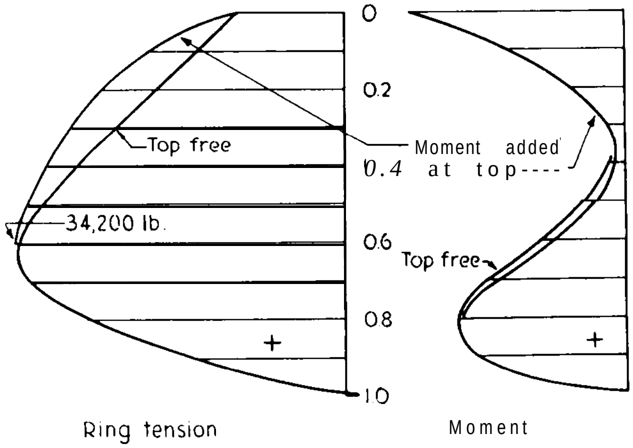


FIG. 19

Point	0.0*H*	0.1*H*	0.2*H*	0.3*H*	0.4*H*	0.5*H*	0.6*H*	0.7*H*	0.8*H*	0.9*H*	1.0*H*
Coef., Table VI	0	+11.41	+13.08	+10.28	+ 6.54	+ 3.34	+ 1.21	− 0.05	− 0.59	− 0.86	− 1.04
Ring ten.	0	+5,100	+5,900	+4,600	+2,900	+1,500	+ 500	0	− 300	− 400	− 500
Coef., Table XI	+1.000	+0.572	+0.252	+0.057	−0.037	−0.065	−0.058	−0.040	−0.018	−0.005	0
Mom.	+6 700	+3,800	+1,700	+ 400	− 200	− 400	− 400	− 300	− 100	0	0

It should be noted that ring tension and moment plotted in Fig. 18 are for moment applied at top when base is free. But the relatively small values near the base in Fig. 18 indicate that the results near the top will be practically the same whether the base is hinged or fixed. The fact that ring tension and wall moment created by the moment applied at top diminish so rapidly is due to the ring elements which exert a strong dampening effect.

The ring tension and the moments determined in this section are now added to those in Section 6.

The effect of adding a moment of $M = 6{,}700$ at the top is shown in Fig. 19. The ring tension is increased near the top. This increase may in some instances become so large that it affects the design materially. The moments are, of course, large at the top and are not likely to be ignored, but the more important increase in ring tension may accidentally be overlooked.

In Section 12 it is shown that the moment at the fixed edge of a roof slab with center support, $R = 27$ ft., and a total design load of $650 - 432 = 218$ lb. per sq.ft.* equals -7,800 ft.lb. per ft. of periphery.

Point	0.0*H*	0.1*H*	0.2*H*	0.3*H*	0.4*H*	0.5*H*	0.6*H*	0.7*H*	0.8*H*	0.9*H*	1.0*H*
Ring ten., Sec. 9	0	+ 5,100	+ 5,900	+ 4,600	+ 2,900	+ 1,500	+ 500	0	− 300	− 400	− 500
Ring ten., Sec. 6	+11,100	+15,200	+19,400	+23,800	+28,000	+31,500	+33,700	+32,700	+27,200	+16,000	0
Total ring ten.	+11.100	+20.300	+25,300	+28,400	+30,900	+33,000	+34,200	+32,700	+26,900	+15,600	− 500
Mom., Sec. 9	+ 6,700	+ 3,800	+ 1,700	+ 400	− 200	− 400	− 400	− 300	− 100	0	0
Mom., Sec. 6	0	0	0	+ 100	+ 500	+ 1,300	+ 2,600	+ 4,200	+ 5,200	+ 4,600	0
Total mom.	+ 6,700	+ 3,600	+ 1,700	+ 500	+ 300	+ 900	+ 2,200	+ 3,900	+ 5,100	+ 4,600	0

*Weight of the roof slab and earth cover minus the surpressure on the stored liquid.

This value is used for determination of moment transmitted from the slab through the joint into the top of the wall.

The procedure is so much like moment distribution applied to continuous frames that the explanation may be brief. The data in Tables XVIII and XIX are stiffnesses which denote moments required to impart a unit rotation at the edge of the wall and the slab. Only relative values of stiffness are required in this application.

The moment required to rotate the tangent at the edge through a given angle is proportional to the following relative stiffness factors.

For wall (Table XVIII for $H^2/Dt = 6$):
$0.783t^3/H = 0.783 \times 15^3/20 = 132$

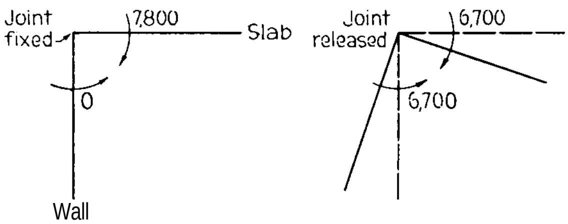
For slab (Table XIX for $c/D = 0.15$):
$0.332t^3/R = 0.332 \times 12^3/27 = 21$

The distribution factors are

$$\text{For wall: } \frac{132}{132 + 21} = 0.86$$

$$\text{For slab: } \frac{21}{132 + 21} = 0.14$$

The dimensions used for the slab are the same as in Section 12.



(a) Fixed end moments (b) Final moments

FIG. 20

The moment of -7,800 ft.lb. tends to rotate the fixed joint as shown in Fig. 20(a). When the artificial restraint is removed, the rotation of the joint will induce new moments in wall and slab. The sums of the induced moments and the original fixed end moments are the final moments. They must be equal but opposite in direction as indicated in Fig. 20(b). The calculations may be arranged in accordance with the usual moment distribution procedure.

	Wall	Slab
Distribution factor	0.86	0.14
Fixed end moment	0	− 7,800
Induced moment (distributed moment)	+ 6,700	+ 1,100
Final moment	+ 6,700	− 6,700

The induced moments equal -7,800 times the distribution factors and are recorded with signs opposite to that of the fixed end moment (unbalanced moment). Note that the wall stiffness is more than six times that of the slab.

Section 10. Wall with Moment Applied at Base

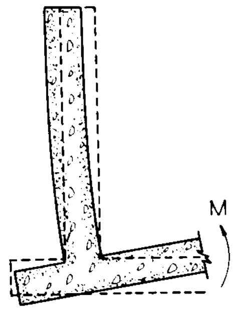


FIG. 21

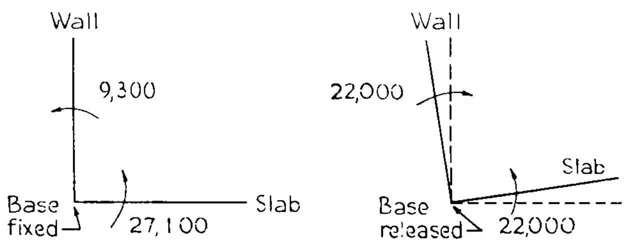
In Sections 4 through 9, the wall has been assumed to rest on a footing not continuous with the bottom slab. The condition to be investigated in this section is illustrated in Fig. 21, in which the wall is made continuous with a reinforced bottom slab designed for uplift.

The design of the slab is discussed in Section 13 in which it is shown that the moment at the fixed edge is -27,100 ft.lb. per ft. No surpressure on the liquid is considered in computing this moment and, therefore, it must also be disregarded in the design of the wall. Accordingly in this section, only triangular load is considered, but if the slab had been designed for surpressure, trapezoidal load should be used for the wall design.

The moment at the base of the wall is first computed on the assumption that the edge is fixed, and a correction is then made for rotation of the edge. The fixed end moment at base of wall is determined for the triangular loading in Section 4 with coefficients selected from Table VII for $H^2/Dt = 6$. Its value is

$$\text{Mom.} = -0.0187 \times wH^3 = -0.0187 \times 62.5 \times 20^3$$
$$= -9{,}350, \text{ say, } -9{,}300 \text{ ft.lb. per ft.}$$

As long as the base is artificially fixed against any rotation, it is subject to two moments both of which tend to rotate the joint in the same direction as shown in Fig. 22(a). One moment is due to the outward pressure of the liquid, the other due to the upward reaction from the subgrade. The base joint is not in equilibrium and when the artificial restraint is removed, it will rotate. The rotation induces moments in wall and slab, and the induced moments added to



(a) Fixed end moments (b) Final moments

FIG. 22

the original fixed end moments must be of such a magnitude that the combined moments are equal but of opposite direction as indicated in Fig. 22(b). Calculation of the final moments may be arranged in accordance with the usual moment distribution procedure.

	Wall	Slab
Distribution factor (same as in Section 9)	**0.86**	**0.14**
Fixed end moment	− 9,300	**− 27,100**
Induced moment (distributed moment)	**+ 31,300**	+ 5,100
Final moment	**+ 22,000**	**− 22,000**

The induced moments, often denoted as distributed moments, are computed by multiplying the "unbalanced moment", 9,300 + 27,100 = 36,400, by the distribution factors. The fixed end moments are recorded with the same sign, negative, since they have the same direction. The induced moments both have positive signs.

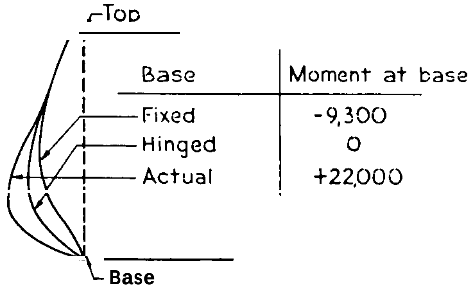


FIG. 23

First, assume the base fixed; and second, apply a moment of 9,300 + 22,000 = 31,300 ft.lb. per ft. of the base. Finally, combine the results of the two steps. The triangular loading is the same as in Section 4, and the value of H^2/Dt = 6 is the same as before. For ring tension, multiply coefficients by wHR = 33,750 lb. per ft. (triangular), and by MR/H^2 = **31,300** $\times$ $27/20^2$ = **2,110** lb. per ft. (*M* at base).

Point	0.0H	0.1*H*	0.2*H*	0.3*H*	0.4*H*	0.5*H*	0.6*H*	0.7*H*	0.8*H*	0.9*H*	1.0*H*
Coef., Table I. Tri.	+0.018	+0.119	+0.234	+		.504	+ .514	+ 0.447		0.112	0
Coef., Table VI. *M* at base	− 1.04	− 0.86	+0.23+7,9(+11,600		+14,900	+17,+3.34	+3,86.54	+ 10.28	+10,2+27	+241.41	0
Ring ten. Tri.	+ 600	+4,000	+7,900	+		,000	+ ,400	+15,100		3,800	0
Ring ten. *M* at base	−2,200	−1,800	−1,200	−		,000	+ ,800	+21,700		4,100	
Total ring ten. Actual	-1.600	+2,200	+6,700	+11,500	+17,500	+24,000	+31.200	+36.800	+37.800	+27.900	0

The rotation of the base and the consequent distribution of moment reveal a significant fact. The change in moment is from -27,100 to -22,000 in the

For moments in a vertical strip, 1 ft. wide, multiply by wH^3 = 500,000 ft.lb. per ft. (triangular), and by M = 31,300 ft.lb. per ft. (*M* at base).

Point	0.0*H*	0.1*H*	0.2*H*	0.3*H*	0.4*H*	0.5*H*	0.6*H*	0.711	0.8*H*	0.9*H*	1.0*H*
Coef., Table VII. Tri.	0	+0.0001	+0.0003	+0.0008	+0.0019	+0.0032	+0.0046	+0.0051	+0.0029	−0.0041	-0.0187
Coef., Table XI. *M* at base	0	− 0.005	− 0.018	− 0.040	− 0.058	− 0.065	− 0.037	+ 0.057	+ 0.252	+ 0.572	+ 1.000
Mom. Tri. Fixed	0	+ 100	+ 200	+ 400	+ 1,000	+ 1,600	+ 2,300	+ 2,600	+ 1,500	− 2,000	− 9,300
Mom. *M* at base	0	− 200	− 600	− 1,300	− 1,800	− 2,000	− 1,200	+ 1,800	+ 7,900	+17,900	+31,300
Total mom. Actual	0	− 100	− 400	− 900	− 800	− 400	+ 1,100	+ 4,400	+ 9,400	+15,900	+22,000

slab but from -9,300 to +22,000 in the wall. For the wall, the effects of three conditions of restraint at the base are shown diagrammatically in Fig. 23. The actual condition is not between fixed and hinged but is far beyond the hinged base assumption. Since the distance between the straight line and the deflection curves in Fig. *23* represents the magnitude of ring tension, it is obviously unsafe to base the design on hinged and especially on fixed-base assumptions.

The wall will now be investigated in two steps.

Ring tension and moments both for fixed base and for actual base condition are plotted in Fig. 24.

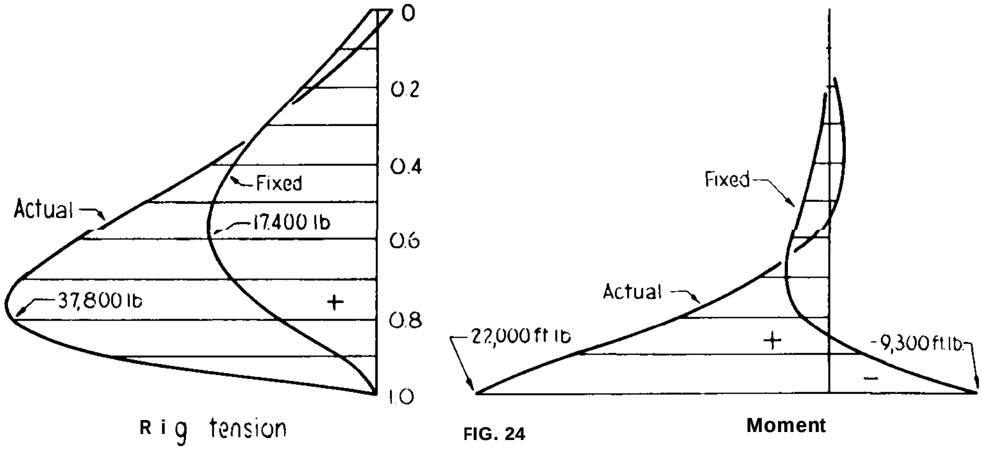


FIG. 24

The maximum ring tension is 17,400 if the base is fixed; but actually it is approximately 37,800 lb., an increase of 117 per cent. Moment at the base is changed from $-9{,}300$ ft.lb. (tension in inside) to $+22{,}000$ ft.lb. (tension in outside). It is clear that continuity between wall and bottom slab materially affects both ring tension and moments. It must be considered in the design.

Shear at base of wall when the base is fixed may be computed as the sum of the products of coefficients taken from Table XVI multiplied by $wH^2 = 62.5 \times 20^2$ = 25,000 lb. per ft. (triangular), and $M/H = 31{,}300/20$ = 1,565 lb. per ft. (M at base).

When base is fixed:

$0.197 \times wH^2 = 0.197 \times 25{,}000 = +\ 4{,}930$ lb.

Effect of M at base:

$-\ 4.49\ M/H = -\ 4.49 \times 1{,}565 = -\ 7{,}020$ lb.

Shear with base released: $V = -\ 2{,}090$ lb.

The tensile stress on the transformed section in Section 6 (A_s = 2.34 sq.in.) is

$$f_c = \frac{CE_sA_s + T_{max.}}{A_c + nA_s} = \frac{21{,}100 + 37{,}800}{15 \times 12 + 10 \times 2.34} = 289 \text{ p.s.i.}$$

Section 11. Roof Slab without Center Support

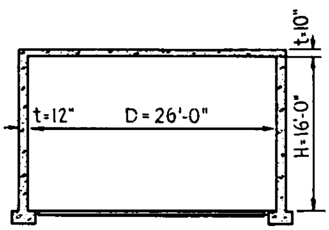


FIG. 25

The application of data for design of roof slabs without interior support is illustrated for the tank sketched in Fig. 25 which carries a superimposed load of 500 lb. per sq.ft. of roof area. The diameter of 54 ft. used in other sections is too large for economical design of this roof slab without center support, so the dimensions in Fig. 25 have been substituted. The total design load is $p = 500 + 125 = 625$ lb. per sq.ft.

For the wall, $H^2/Dt = 16.0^2/26.0 \times 1.0 = 9.8$, say, 10. From Table XVIII, for $H^2/Dt = 10$, the relative stiffness of the wall is $1.010t^3/H = 1.010 \times 12^3/16 = 109$. The relative stiffness of a circular plate without interior support (from Table XIX) is $0.104t^3/R = 0.104 \times 10^3/13$ = 8.0. The relative values computed suffice for the calculation of distribution factors which are

$$\text{For wall: } \frac{109}{109 + 8} = 0.93$$

$$\text{For slab: } \frac{8}{109 + 8} = 0.07$$

When the slab is considered fixed at the edge, the edge moment may be computed by multiplying pR^2 by the coefficient from Table XII at Point $1.00R$: $-0.125 \times pR^2 = -0.125 \times 625 \times 13^2 = -13{,}200$ ft.lb. per ft. of periphery. Inside diameter is used for all calculations here. Actually, a somewhat larger value should be used for some of the calculations, but proportioning of the slab should be made at inside face of wall.

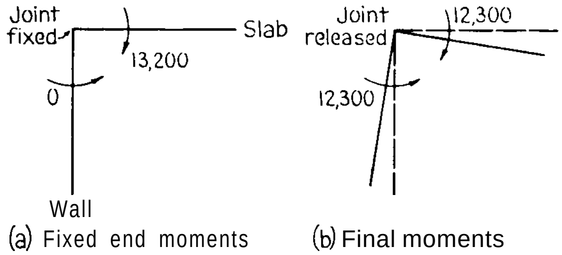


FIG. 26

The procedure in determining the final moment at the edge has already been illustrated in Sections 9 and 10. The fixed end moments at the edge of the slab in this section are shown in Fig. 26(a), and the final moments in Fig. 26(b) are computed below by the ordinary moment distribution procedure.

	Wall	Slab
Distribution factor	0.93	0.07
Fixed end moment	0	− 13,200
Induced moment (distributed moment)	+ 12,300	+ 900
Final moment	+ 12,300	− 12,300

It is seen that a large moment is induced in the top of the wall. It has been shown in Section 9 how to determine ring tensions and moments in a wall caused by a moment at top of the wall. The slab only is discussed in this section.

$$\text{Shear: } V = \frac{\pi pR^2}{2\pi R} = \frac{pR}{2} = \frac{625 \times 13}{2} = 4{,}060 \text{ lb. per ft.}$$

$$\text{Unit shear: } v = \frac{V}{0.875bd} = \frac{4{,}060}{0.875 \times 12 \times 8.5} = 45 \text{ p.s.i.}$$

The roof slab in Fig. 25 is first assumed to be fixed and a correction is then added for the effect of a moment applied at the edge. For illustration, consider a tank in which the joint at top of wall is discontinuous so the slab may be assumed to be hinged. The moments in the hinged slab may be computed by determining moments in a fixed slab, using coefficients in Table XII, and adding to them the moments in a slab in which an edge moment of $0.125pR^2$ ft.lb. per ft. is applied. The most convenient way to do this is to add 0.125 to all the coefficients in Table XII, both for radial and tangential moments, and then to multiply the modified coefficients by pR^2. Note that the coefficient for radial moment at the edge becomes zero by the addition of 0.125, and the tangential moment becomes 0.100. These are the values for a slab hinged at the edge.

In this problem the moment induced at edge of slab equals + 900 ft.lb. per ft. Therefore, the final moment coefficients are those for fixed edge in Table XII to each of which must be added a quantity equal to $+900/pR^2 = +900/625 \times 13^2 = +0.009$. The coefficients and moments are as follows, Point 0.0R denoting the center and Point 1.0R the edge of slab. Multiply coefficients by $pR^2 = 625 \times 13^2 = 105{,}600$ ft.lb. per ft.

Point	0.0R	0.1R	0.2R	0.3R	0.4R	0.5R	0.6R	0.7R	0.8R	0.9R	1.0R
Coef., Table XII. Rad.	+0.075	+0.073	+0.067	+0.057	+0.043	+0.025	+0.003	−0.023	−0.053	−0.087	− 0.125
Add + 0.009	+0.084	+0.082	+0.076	+0.066	+0.052	+0.034	+0.012	−0.014	−0.044	−0.078	− 0.116
Coef., Table XII. Tang.	+0.075	+0.074	+0.071	+0.066	+0.059	+0.050	+0.039	+0.026	+0.011	−0.006	− 0.025
Add + 0.009	+0.084	+0.083	+0.080	+0.075	+0.068	+0.059	+0.048	+0.035	+0.020	+0.003	− 0.016
Rad. mom. per ft.	+8,900	+8,700	+8,000	+7,000	+5,500	+3,600	+1,300	−1,500	−4,700	−8,200	−12,300
Tang. mom. per ft.	+8,900	+8,800	+8,500	+7,900	+7,200	+6,200	+5,100	+3,700	+2,100	+ 300	− 1,700
Rad. mom. per seg.*	0	+ 900	+1,600	+2,100	+2,200	+1,800	+ 800	−1,100	−3,800	−7,400	−12,300

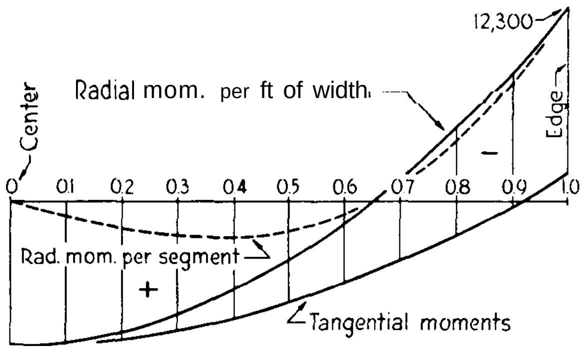


FIG. 27

The solid-line curves in Fig. 27 are for moments per ft. of width. The dash line indicates radial moments for a segment that is 1 ft. wide at the edge. Values on the dash line are obtained by multiplying the radial moment per ft. by the fraction indicating its distance from the center. For example, multiply 12,300 by 1.0; 8,200 by 0.9; 4,700 by 0.8; and so forth.

The maximum negative moment is 12,300 ft.lb. per ft.

$$A_s = \frac{M}{ad} = \frac{12.3}{1.44 \times 8.5} = 1.00 \text{ sq.in}$$

Use 1-in. round bars spaced $9\frac{1}{2}$ in. *o.c.* (A_s = 1.00) in top of slab and outside of wall at corner. Total number required is $2\pi R/9.5 = 2\pi \times 13 \times 12/9.5 = 103$, say, 104 bars.

From Table 4 (*Handbook***), for $bd = 12 \times 8\frac{1}{2}$: $F = 0.072$, and $K = M/F = 12.3/0.072 = 171$. $K = 236$ is allowed for $f_s/n/f_c' = 20{,}000/10/3{,}000$.

It is seen from the dash line in Fig. 27 that one-half of the 104 top bars may be discontinued at a distance from the inside of the wall equal to 0.13R + 12 diameters = $0.13 \times 13 + 12 \times 1.0/12 = 1.69 + 1.00 = 2.69$ ft., say, 2 ft. 9 in. The other 52 top bars may be discontinued at a distance of 0.37R + 12 diameters = $0.37 \times 13 + 12 \times 1.0/12 = 4.8 + 1.0 = 5.8$ ft., say, 5 ft. 10 in. from the inside of the wall. All these bars are placed radially.

The largest number of radial bars for positive moments is between Points 0.3R and 0.4R where the dash line has its maximum value. At Point 0.4R, the moment is 5,500 ft.lb. per ft., and the length of the concentric circle through 0.4R is $2\pi(0.4R) = 2\pi \times 0.4 \times 13 = 32.7$ ft.

At Point 0.4R: $A_s = \frac{32.7M}{ad} = \frac{32.7 \times 5.5}{1.44 \times 9.0} = 13.9$ sq.in.

Use thirty-two $\frac{3}{4}$-in. round bars ($A_s = 14.08$).

The dash line in Fig. 27 shows that the radial moment per segment converges toward zero at the center. Actually, most of the radial bars must be extended close to or across the center.

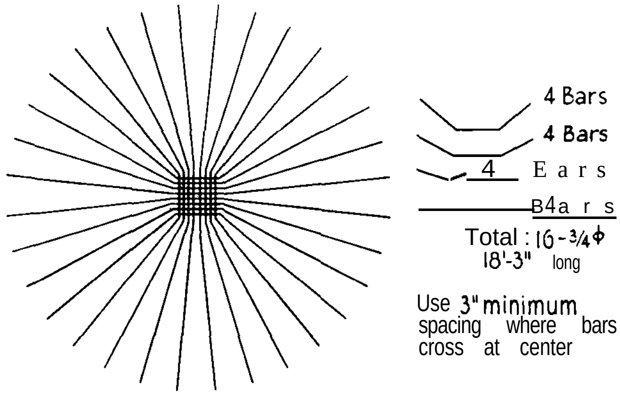


FIG. 28

Fig. 28 shows one arrangement with eight radial bars in each quadrant. Sixteen bars, 18 ft. 3 in. long, are required for the whole slab and are bent as shown, the minimum spacing at center being approximately 3 in. If desired, some of the bars in Fig. 28 may be discontinued in accordance with the steel requirements represented by the dash line in Fig. 27. Note that there are only two layers where the bars cross at center in Fig. 28 and that only four types of bent bars are required.

Ring bars are proportioned so as to fit the tangential moment curve in Fig. 27. The radius of the smallest ring bar may be 1 ft. Maximum area is required near the center and equals $A_s = \frac{M}{ad} = \frac{8.9}{1.44 \times 8.5} = 0.73$ sq.in. Use $\frac{7}{8}$-in. round bars spaced 10 in. o.c.

Ring bar areas decrease gradually toward Point 0.9R. Inside this point, the bars are all in the bottom, but outside, they are in the top. Laps may be spliced in accordance with code requirements, or the joints may be welded.

*1 ft. wide at edge.

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Section 12. Roof Slab with Center Support

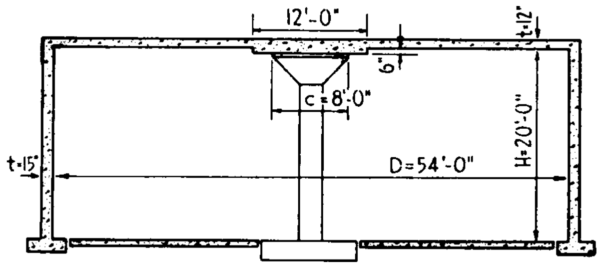

FIG. 29

In this section the original tank dimensions given in Sections 4 through 10 will be used. The top slab is as sketched in Fig. 29. It is designed for a superimposed load of 500 lb. per sq.ft. Its thickness is 12 in., and it has a drop panel with 6-in. depth and 12-ft. diameter. The capital of the column has a diameter of c = 8 ft. Slab and wall are assumed to be continuous.

Data are presented in Tables XIII, XIV and XV for slabs with center support for the following ratios ofcapital to wall diameter: c/D = 0.05, 0.10, 0.15, 0.20, and 0.25. The tables are for fixed and hinged edge as well as for a moment applied at the edge.

The general procedure in this section is the same as in Section 11. First consider the edge fixed and compute fixed end moments. Then, distribute moments at the edge, and finally, make adjustments for the change in edge moment.

All the table values are based on a uniform slab thickness. Adding the drop panel will have some effect, but it is believed that the change is relatively small especially since the ratio of panel area to total slab area is as small as 1:20.

The relative stiffness factors are 0.86 for the wall and 0.14 for the slab (*see* Section 9).

The radial fixed end moment equals the coefficient of -0.0490 from Table XIII (for c/D = 8/54 = 0.15 at Point 1.0R) multiplied by pR^2. Two values of p will be considered. For the slab, use p = 650, which gives -0.0490 X 650 X 27^2 = -23,200 ft.lb. per ft. When there is a surpressure on the liquid in the tank of 432 lb. per sq.ft., the combined downward load on the slab is p = 650 − 432 = 218, and the fixed end moment is -0.0490 X 218 X 27^2 = -7,800 ft.lb. per ft.

trapezoidal load distribution. The final edge moment for which the slab is designed is -23,200 (1 − 0.14) = -20,000 ft.lb. per ft.

The procedure is to design the slab for fixed edge (−23,200 ft.lb.), and then add the effect of a moment of 23,200 − 20,000 = 3,200 ft.lb. applied at the edge, but first, shearing stresses are investigated.

The column load is determined by multiplying coefficients taken from Table XVII by pR^2.

When edge is fixed:

1.007 pR^2 = 1.007 X 650 X 27^2 = 478,000 lb.

Effect of moment at edge:

9.29 M = 9.29 X 3,200 = 30,000 lb.

Total column load = 508,000 lb.

Load on concrete in 30-in. round tied column:

0.225 X 3,000 X 0.8 X A_g = 382,000 lb.

Balance: 508,000 − 382,000 = 126,000 lb.

Use ten 1-in. square bars.

Radius of critical section for shear around capital is 48 + 18 − 1.5 = 64.5 in. = 5.37 ft. Length of this section is 2π X 64.5 = 405 in. Load on area within the section is 650 X π X 5.37^2 = 59,000 lb. Unit shear equals

$$v = \frac{V}{0.875bd} = \frac{508{,}000 - 59{,}000}{0.875 \times 405 \times 16.5} = 77 \text{ p.s.i}$$

Radius of critical section for shear around drop panel is 72 + 12 − 1.5 = 82.5 in. = 6.88 ft. Length of this section is 2π X 82.5 = 518 in. Load on area within the section is 650 X π X 6.88^2 = 96,000 lb. Unit shear equals

$$v = \frac{V}{0.875bd} = \frac{508{,}000 - 96{,}000}{0.875 \times 518 \times 10.5} = 87 \text{ p.s.i.}$$

Shear at edge of wall: $V = \pi pR^2$ − column load = π X650 X 27^2 − 508,000 = 1,489,000 − 508,000 = 981,000 lb. Unit shear is

$$v = \frac{V}{0.875bd} = \frac{981{,}000}{0.875 \times \pi \times 2 \times 27 \times 12 \times 10.5}$$
= 52 p.s.i.

The radial moments are computed by selecting coefficients for c/D = 0.15 from Tables XIII and XV, and multiplying them by $pR^2 = 650 \times 27^2 = 474{,}000$ ft.lb. per ft. (for fixed edge), and by M = 3,200 ft.lb. per ft. (for moment at edge).

Point	0.15R	0.2R	0.25R	0.3R	0.4R	0.5R	0.6R	0.7R	0.8R	0.9R	1.0R
Coef., Table XIII. Fixed	−0.1089	−0.0521	−0.0200	+0.0002	+0.0220	+0.0293	+0.0269	+0.0169	+0.0006	−0.0216	−0.0490
Coef., Table XV. M at edge	− 1.594	− 0.930	− 0.545	− 0.280	+ 0.060	+ 0.323	+ 0.510	+ 0.663	+ 0.790	+ 0.900	+ 1.000
Rad. mom. Fixed	−51,600	−24,700	− 9,500	+ 100	+10,400	+13,900	+12,800	+ 6,000	+ 300	−10,200	−23,200
Rad. mom. M at edge	− 5,100	− 3,000	− 1,700	− 900	+ 200	+ 1,000	+ 1,600	+ 2,100	+ 2,500	+ 2,900	+ 3,200
Total rad. mom., per ft.	−56,700	−27,700	−11,200	− 800	+10,600	+14,900	+14,400	+10,100	+ 2,800	− 7,300	−20,000
Total rad. mom., per seg.	− 8,500	− 5,500	− 2,800	− 200	+ 4,200	+ 7,500	+ 8,600	+ 7,100	+ 2,200	− 6,600	−20,000

This is used as basis for the moment distribution in Section 9 which results in a final edge moment of -7,800 (1 − 0.14) = -6,700. The wall is designed for this moment with opposite sign combined with a

Radial moments in the last line are for a segment having an arc 1 ft. long at the edge (Point 1.0R). They are obtained by multiplying the original moment per ft. by the fraction indicating its distance from the center. For illustration: 14,400 X 0.6 = 8,600 ft.lb.

The moments in the two last lines of the table are plotted in Fig. 30. The maximum negative moment at the center occurs at the edge of the column capital. The circumference of the capital is 8π ft., and the total maximum negative moment around the edge is 56,700 X 8π = 1,425,000 ft.lb.

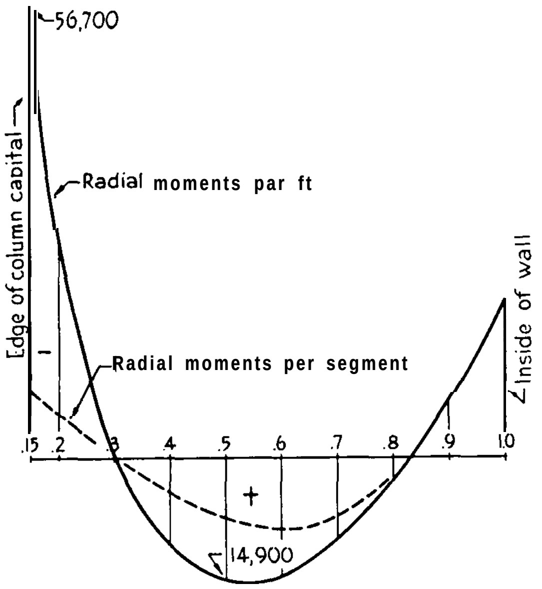

FIG. 30

The theoretical moment across the section around the capital is larger than the moment that actually exists. It should be remembered that the moment coefficients in this section are computed for a slab that is assumed to be fixed at the edge of the capital. Actually, the edge is not fixed, but it has some rotation and a reduction in the theoretical moment results.

The problem of determining the actual moment at the capital is similar to that which exists in regular flat slab design. As a matter of fact, the region around the center column in the tank slab is stressed very much as in ordinary flat slab floor construction, so that the design should be practically identical in the column region of both types of structures.

Westergaard* has worked out moments in flat slab in terms of the quantity: $0.125WL\,(1 - 2c/3L)^2$. In all modern codes, however, the coefficient of 0.125 is replaced by 0.09, a reduction of 28 per cent. Other adjustments made in such codes introduce still greater reductions in some of the theoretical moments at the column capital. Such modified design moments have been thoroughly investigated by numerous test loadings of flat slab floors and are generally accepted for use in design.

In view of the facts discussed, it seems reasonable and conservative to allow a 28 per cent reduction in the theoretical moments around the center column of the tank slab. The reduction will be used here for radial moments at the capital only. Tangential moments at capital could probably be reduced also, but they are already comparatively small even without the reduction.

For the slab in Fig. 29 the total moment around the edge of the capital will then be taken as (1 − 0.28) X 1,425,000 = 1,026,000 ft.lb. The steel area is

$$A_s = \frac{M}{ad} = \frac{1{,}026}{1.44 \times 16.5} = 43.2 \text{ sq.in.}$$

Use twenty-eight 1x-in. square bars (A_s = 43.68) and arrange the bars in top of the slab as in Fig. 31.

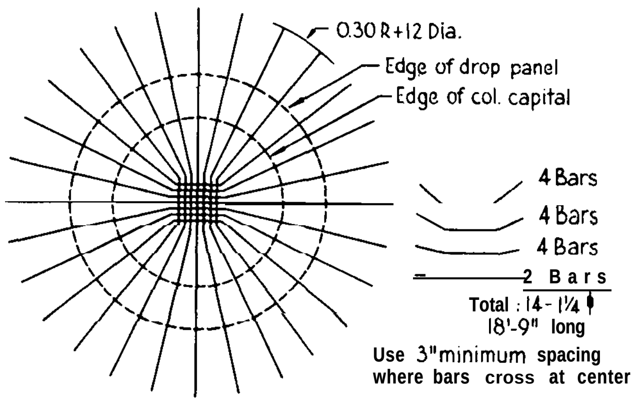

FIG. 31

Across the edge of the drop panel the moment is 20,000 ft.lb. per ft. at Point $(6/27)R = 0.22R$, or

$$M = 12\pi \times 20{,}000 \times (1 - 0.28) = 543{,}000 \text{ ft.lb.}$$

$$A_s = \frac{M}{ad} = \frac{543}{1.44 \times 10.5} = 35.9 \text{ sq.in.}$$

The twenty-eight 1x-in. square bars are ample.

Positive moment per segment is maximum at Point $0.6R$ as indicated by the dash-line curve in Fig. 30. The total moment at this point is

$$M = 14{,}400 \times 2\pi \times 0.6 \times 27 = 1{,}465{,}000 \text{ ft.lb.}$$

$$A_s = \frac{M}{ad} = \frac{1{,}465}{1.44 \times 10.5} = 97 \text{ sq.in.}$$

Use one hundred sixty ⅞-in. round bars (A_s = 96).

$$\text{Spacing at Point } 0.6R \text{ is } \frac{2\pi \times 0.6 \times 27 \times 12}{160} = 7.6 \text{ in.}$$

Positive reinforcement may be discontinued at points 12 diameters beyond sections 0.30 X 27 = 8.1 ft. and 0.83 X 27 = 22.4 ft. from the center as shown bv the curves in Fig. 30. The total over-all length of positive reinforcement is

$$22.4 - 8.1 + 2 \times \tfrac{7}{8} = 16.0 \text{ ft.}$$

If some of these bars are to be made shorter than 16 ft., use the dash-line curve in Fig. 30 for determining where bars can be discontinued.

*"Moments and Stresses in Slabs", *Proceedings of American Concrete Institute*, 1921, pages 415-538.

Maximum negative moment at inside of wall is 20,000 X 27r X 27 = 3,390,000 ft.lb.

$$A_s = \frac{M}{ad} = \frac{3{,}390}{1.44 \times 10.5} = 224 \text{ sq.in.}$$

Use two hundred eighty-four l-in. round bars (A_s = 224.36). Spacing at the wall is

$$\frac{2\pi \times 27 \times 12}{284} = 7.2 \text{ in.}$$

All of these bars may be discontinued at a distance from inside of wall equal to 0.17 X 27 + 12 diameters = 4.6 + 1.0 = 5.6 ft., say, 5 ft. 7 in.

The tangential moments are computed by selecting coefficients for c/D = 0.15 from Tables XIII and XV, and multiplying them by pR^2 = 474,000 ft.lb. per ft. (for fixed edge), and by M = 3,200 ft.lb. per ft. (for moment at edge).

Point	0.15*R*	0.2*R*	0.258	0.3*R*	0.4*R*	0.5*R*	0.6*R*	0.7*R*	0.8*R*	0.9*R*	1.0*R*
Coef., Table XIII. Fixed	-0.0218	-0.0284	-0.0243	-0.0177	-0.0051	+0.0031	+0.0080	+0.0086	+0.0057	-0.0006	-0.0098
Coef., Table XV. *M* at edge	- 0.319	- 0.472	- 0.463	- 0.404	- 0.251	- 0.100	+ 0.035	+ 0.157	+ 0.263	+ 0.363	+ 0.451
Tang. mom. Fixed	-10,300	-13,500	-11,500	- 8,400	- 2,400	+ 1,500	+ 3,800	+ 4,100	+ 2,700	- 300	- 4,600
Tang. mom. *M* at edge	- 1,000	- 1,500	- 1,500	- 1,300	- 800	- 300	+ 100	+ 500	+ 800	+ 1,200	+ 1,400
Total tang. mom., per ft.	-11,300	-15,000	-13,000	- 9,700	- 3,200	+ 1,200	+ 3,900	+ 4,600	+ 3,500	+ 900	- 3,200

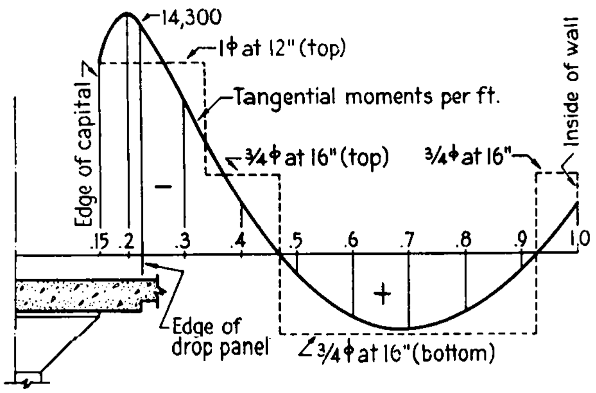

FIG. 32

The total tangential moments are plotted in Fig. 32. Within the drop panel, the effective depth is 16.5 in. instead of 10.5 in., and if the moments in that region are reduced in the ratio of 10.5/16.5, it is seen that the critical moment, 14,300 ft.lb. per ft., occurs at rhe edge of the drop panel. The maximum steel area is

$$A_s = \frac{M}{ad} = \frac{14.3}{1.44 \times 10.5} = 0.95 \text{ sq.in. per ft.}$$

Use seven l-in. round bars spaced 12 in. o.c. Place the first bar at the edge of capital, and the seventh bar at a distance of 9 ft. from the center. In the rest of the slab, all the way out to the wall, use fourteen ¾-in. round bars spaced 16 in. o.c. As indicated in Fig. 32, some of the bars are in the top, others in the bottom of the slab, depending on the sign of the tangential moments. The bars are circular and are either lap-spliced or welded.

Section 13. Base Slab with Center Support

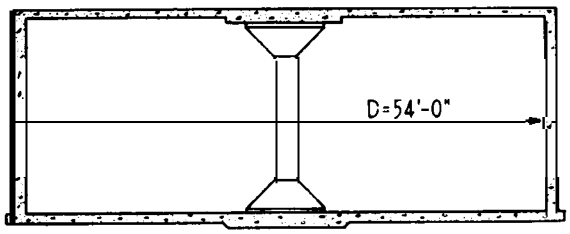

FIG. 33

When the bottom of the tank is below ground water level, the upward hydrostatic pressure on the bottom must be investigated. If the upward pressure exceeds the dead load of the tank floor, there may be danger of heaving unless the floor is built and reinforced as a structural slab similar to that used for the roof, but with the loading directed upward rather than downward.

The loading on the bottom when a tank like that shown in Fig. 33 is empty equals the load on the roof plus the weight of wall and column. In this example, the same data as in Section 12 are used. The total roof load is 650 lb. per sq.ft., the outside diameter of the roof slab is 56.5 ft., the wall is 15 in. thick and 20 ft. high, and the center column is 30 in. in diameter. The total superimposed load on top of the bottom slab is

Roof:	$650 \times \pi \times 28.25^2$	= 1,630,000 lb.
Wall :	$187.5 \times 20 \times \pi \times 55.25$	= 650,000 lb.
Column:	4.9 X 150 X 20 = 14,700, say,	20,000 lb.
	Total load	= 2,300,000 lb.

With a 9-in. projection, the outside diameter of the bottom slab is 58 ft. If the loading of 2,300,000 lb. is assumed to be distributed uniformly over the subgrade, the upward reaction on the bottom slah will he

$$P = \frac{2{,}300{,}000}{\pi \times 29^2} = 870 \text{ lb. per sq.ft.}$$

This is 34 per cent more than the design load on the roof. If the same design procedure is used for top and bottom slab, the upward load on the column would be far greater than the downward load. This is, of course, impossible. Under the conditions considered, the assumption of uniform distribution of reaction over the subgrade is obviously in error.

A rigorous treatment of a slab on an elastic foundation is beyond the scope of this discussion, so the design of the bottom slab will here be based on what is believed to be a reasonable estimate. The column load computed in Section 12 is 508,000 lb., so it will be assumed here that the column reaction on both top and bottom slab is 508,000 lb. It therefore seems reasonable to choose the dimensions and reinforcement around the bottom of the column essentially the same as around the top as designed in Section 12. One difference, indicated in Fig. 33, is that the drop panel is kept below the bottom slab. This drop panel may be made large enough to accommodate the rosette of bars shown in Fig. 31 which is to be placed with a 3-in. clearance above the surface of the subgrade.

It seems advisable to make the bars for positive moments-creating tension in the inside of the slab—identical for top and bottom slab, that is, to use one hundred sixty $\frac{7}{8}$-in. round bars 16 ft. long, placed in the top of the 12-in. bottom slab. The position of these bars is the same as that for the top slab indicated by the dash-line curve in Fig. 30.

Negative moments at the wall are probably greater in the bottom than in the top slab. The tendency is for the wall load to be distributed to the subgrade near the wall so that the soil reaction is maximum at the wall. No theoretical solution for the determination of the resulting moment is known, so an arbitrary procedure will be adopted. In the top slab where the load is p = 650, the fixed end moment per ft. from Section 12 is -23,200 ft.lb. In the bottom slab, where p = 870, this negative moment will be increased to:

$$-23\,200 \times \frac{(870 + 650)}{2 \times 650} = -27{,}100 \text{ ft.lb.*}$$

Distributing fixed end moments as in Section 10 gives a final moment of -22,000 ft.lb. per ft. For the whole circumference, the total negative moment is 22,000 X 2π X 27 = 3,730,000 ft.lb.

$$A_s = \frac{M}{ad} = \frac{3{,}730}{1.44 \times 10.5} = 247 \text{ sq.in.}$$

Use three hundred fourteen 1-in. round bars (A_s = 248.06) which may be stopped, say, 6 ft. from the inside face of the wall.

Total shear at inside of wall is

$$V = \pi p R^2 - \text{column load} = \pi \times 870 \times 27^2 - 508{,}000$$
$$= 1{,}995{,}000 - 508{,}000 = 1{,}487{,}000 \text{ lb.}$$

$$v = \frac{V}{0.875bd} = \frac{1{,}487{,}000}{0.875 \times \pi \times 2 \times 27 \times 12 \times 10.5} = 80 \text{ p.s.i.}$$

Due to increase in moment and shear, it may be advisable to deepen the bottom slab at the wall from 12 in. to, say, 18 in. It may possibly be better to use the drop panel depth of 18 in. all over the bottom slab and to reduce the amount of reinforcing steel. The extra weight of concrete does not add to the moments in the bottom slab as it would in the top slab.

Section 14. Roof Slab with Four Interior Supports

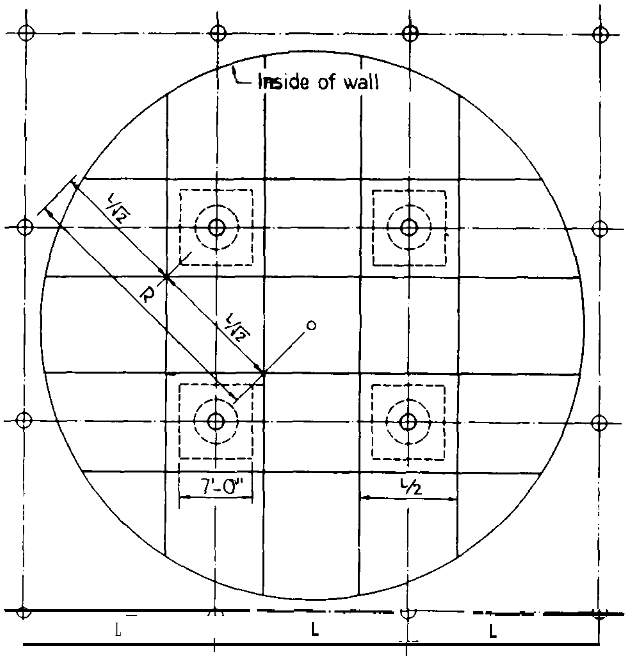

FIG. 34

Fig. 34 illustrates a column layout for a regular flat slab floor with span length, L, on which a circle with radius, R, is superimposed representing the inside face of a tank wall. The circle in Fig. 34 is chosen so that the columns are midway between the wall and the center of the tank which gives $L = R/\sqrt{2}$.

Based on the radius of $R = 27$ ft. used in preceding sections, $L = 27/\sqrt{2} = 19.1$, say, 19 ft. The tank slab is designed as if it were a regular flat slab floor consisting of interior panels and the general procedure will be in accordance with the A.C.I. Code 1956.

Estimate 9 in. for slab thickness which gives load on roof equal to

Live load:	100
Earth:	400
Slab:	112
Total:	612 lb. per sq.ft.

The sum of the positive and negative moments at the principal design sections of a flat slab panel is

$$M_0 = 0.09WL\left(1 - \frac{2c}{3L}\right)^2$$

W, the total load on a panel, equals pL^2 = 612 X 19^2 = 221,000 lb.

*This moment is not changed by filling the tank since the weight of the liquid is transmitted directly to the subgrade without creating any moments. But surpressure acts on both top and bottom and creates moments in slabs and walls. It tends to reduce all moments in slabs and will therefore bc disregarded here.

c, the diameter of the column capital, is 4.5 ft.

L, the span length of a typical square panel, is 19 ft. Then

$$M_o = 0.09 \times 221{,}000 \times 19.0\left(1 - \frac{2\times 4.5}{3\times 19.0}\right)^2 = 268{,}000 \text{ ft.lb.}$$

This moment is to **be** prorated to the principal design sections in accordance with Table 1004(f) in the A.C.I. Code **1956;** but first shear is investigated on the assumption that the column **load** equals $W = 221{,}000$ lb.

Radius to critical section of shear around capital, where the depth of slab plus drop panel equals 9 + 4.5 = 13.5 in., is 27 + 13.5 − 1.5 = 39 in. = 3.25 ft. The shear on this section is 221,000 − a X 3.25^2 X 612 = 221,000 − 20,000 = 201,000 lb. The length of the section is 2π X 39 = 245 in. The unit shear equals

$$v = \frac{V}{0.875bd} = \frac{201{,}000}{0.875 \times 245 \times 12} = 78 \text{ p.s.i}$$

Distance to critical section of shear around square drop panel is 42 + 9 − 1 = 50 in. = 4.17 ft. The shear is 221,000 − 4 X 4.17” X 612 = 178,000 lb. Length of section is 8 X 50 = 400 in. and unit shear equals

$$v = \frac{V}{0.875bd} = \frac{178{,}000}{0.875 \times 400 \times 8} = 64 \text{ p.s.i.}$$

Bending moments at principal design sections in an interior flat slab panel are recorded below. The tensile steel area, A_s, is computed by dividing moments in ft.kips by $1.44d$, the factor of 1.44 being taken from Table 1 for f_s = 20,000 in *Reinforced Concrete Design Handbook.*

The number and size of bars are those required in a typical interior flat slab panel. They may also be used in the exterior panels of the roof slab in Fig. 34, but the bar detailing must be modified to suit the circular shape of the slab edge.

tied to the botrom of the radial bars.

In the absence of more exact data, it will be necessary to estimate the design load for columns and footings. The panel load, W = 221,000 lb., is evidently larger than the actual column load as will be seen by inspection of the areas contributing to each column in Fig. 34. By considering separately the reactions reaching a column from each adjacent quadrant, it appears reasonable to set the column load equal to

$$\frac{W}{4}(1 + 1 + 1 + \tfrac{1}{2}) = \frac{221{,}000}{4} \times 3.5 = 194{,}000 \text{ lb.}$$

Deducting four column loads from total load on slab gives total shear around the periphery.

Section 15. Roof Slab Designed as Three-Way Flat Slab

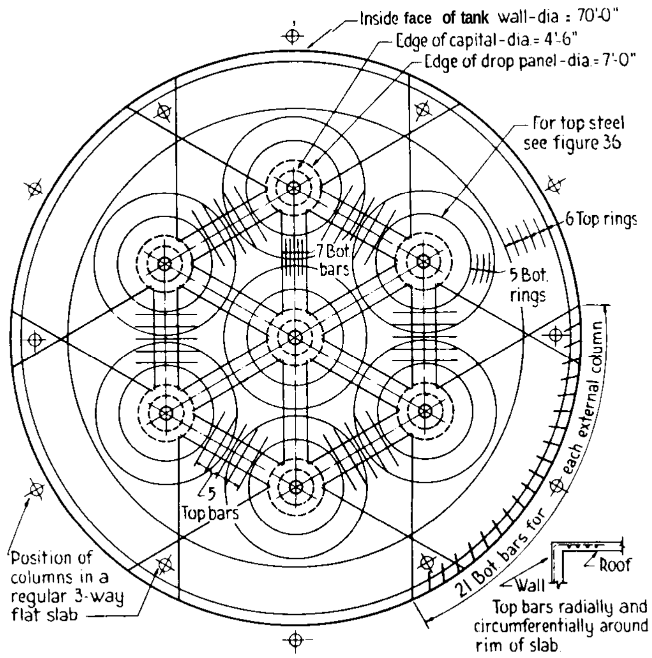

FIG. 35

	Mom. ft.kips	d in.	A_s	Reinforcement	Description
Col. strip, neg.	$0.50M_0$ = 134	12.0	7.8	26 5/8-in. rd.	10 str. top. 16 bt.
Col. strip, pos.	$0.20M_0$ = 54	8.0	4.7	16 5/8-in. rd.	Alternate bars bent
Middle strip, neg.	$0.15M_0$ = 40	8.0	3.5	18 1/2-in. rd.	18 bt. from adj. spans
Middle strip, pos.	$0.15M_0$ = 40	8.0	3.5	18 1/2-in. rd.	Alternate bars bent

The negative radial moment, assuming the edge of the slab tixed, equals approximately $0.025pR^2$ ft.lb. per ft. of periphery*. Actually, the edge is continuous but not necessarily fixed. The relative stiffnesses of slab and wall are not known, but if the ratio of 1:4 is assumed, the radial edge moment will be equal to

$$\frac{4}{1+4} \times 0.025pR^2 = 0.020 \times 612 \times 27^2 = 8{,}900 \text{ ft.lb.}$$

per ft. Radial steel in the top face at the edge is then

$$A_s = \frac{M}{ad} = \frac{8.9}{1.44 \times 8} = 0.77 \text{ sq.in. per ft.}$$

For tangential moments at edge of slab use a nominal reinforcement of two 3/4-in. round circular bars

The four interior columns in Fig. 34 are adequate for a tank diameter of 54 ft. but not for a diameter as large as, say, 70 ft. when the slab must support a tot.11 load of more than 600 lb. per sq.ft. It is then more economical to increase the number of columns. Fig. 35 shows **a layout** with seven columns arranged as in a three-way flat slab.

It is clear that the arrangement of seven columns in Fig. 35 has an advantage in that the six intermediate columns lie on a circle which is concentric with the

*This coefficient was determined in the same way as those in Table XIII.

tank wall. The span length adjacent and perpendicular to the wall is much more uniform with the seven columns than with nine columns arranged as in a regular two-way flat slab floor system.

The following data are used in illustrating the design of the slab.

Superimposed load: 4 ft. of earth + 100-lb. live load = 500 lb. per sq.ft.

Slab: 8 in. deep, which gives total load = 100 + 500 = 600 lb. per sq.ft.

Drop panel: 7-ft. diameter and 4-in. depth.

Column capital: 4 ft. 6 in. diameter.

The distance from the center column to the intermediate columns in Fig. 35 is chosen, approximately 10 per cent greater than the exterior span and will therefore be $0.525R$ = 0.525 X 70/2 = 18.4, say, 18 ft. 6 in. The distance from the intermediate columns to the wall is then 35.0 − 18.5 = 16.5 ft.

The area of the parallelogram between four adjacent columns in Fig. 35 is 18.52 X cos 30° = 18.5^2 X 0.866 = 297 sq.ft. The load per column equals 297 X 600 = 178,000 lb. Use 18-in. round column. Load on concrete = 0.18 X 3,000 X 254 = 137,000 lb. Balance of 41,000 lb. to be carried by longitudinal bars stressed to 0.8 X 16,000 = 12,800 p.s.i. A_s = 41,000/12,800 = 3.20 sq.in. Use four l-in. round bars with circular ties or the lightest possible spiral.

Radius to critical section for shear around capital = 27 + 12 − 1.5 = 37.5 in. = 3.12 ft. Load inside this radius = a X 3.122 X 600 = 18,000 lb., and circumference = 2π X 37.5 = 236 in.

$$v = \frac{V}{0.875bd} = \frac{178{,}000 - 18{,}000}{0.875 \times 236 \times 10.5} = 74 \text{ p.s.i.}$$

Radius to critical section for shear around drop panel = 42 + 8 − 1 = 49 in. = 4.08 ft. Load inside this radius = π X 4.082 X 600 = 31,000 lb., and circumference = 2π X 49 = 308 in.

$$v = \frac{V}{0.875bd} = \frac{178{,}000 - 31{,}000}{0.875 \times 308 \times 7} = 78 \text{ p.s.i.}$$

The three-way slab will be designed on basis of data tabulated for circular slabs with center support and fixed edge. When considering the region around the center column in Fig. 35, it will be assumed that a fixed edge with radius R_1* can be substituted for the six intermediate columns. Determine value of R_1 and design the slab around the center column using the data in Table XIII. For the six intermediate columns, the design made for the center column may be used with certain modifications. While not theoretically correct, this procedure will produce a reasonable and practical design.

The load on a center column supporting a slab with radius R_1 and having fixed edge equals: Coef. X pR_1^2. The coefficient is 0.919 for $c/2R_1$ = 0.10 and 1.007 for *0.15 (see Table XVZZ).* The value of c is 4.5 ft., but R_1 is as yet unknown. Estimate R_1 = 18.0 which gives $c/2R_1$ = 4.5/36.0 = 0.125 and coefficient = ½ X (0.919 + 1.007) = 0.963. The column load in the circular slab with fixed edge equals 0.963 pR_1^2, and in the three-way flat slab, it equals 178,000 lb. Since these loads must be equal, R_1 can be determined from the equation

$$0.963 \times 600 \times R_1^2 = 178{,}000$$

which gives

$$R_1 = 17.6, \text{ say, } 17 \text{ ft. } 6 \text{ in.}$$

Use R_1 = *17.5* ft. and $c/2R_1$ = 0.125 in all subsequent calculations for moments in the slab.

Maximum negative radial moment at column. Coefficient from Table XIII is ½ X (-0.1433 − 0.1089) = -0.126. Mom. per ft. = $-0.126pR_1^2$ = -0.126 X 600 X 17.5^2 = -23,200 ft.lb. Moment along entire circumference of column capital = -23,200 X π X 4.5 = -328,000 ft.lb. As discussed in Section 12, reduce this moment 28 per cent which gives 0.72 X (−328,000) = -236,000 ft.lb.

$$A_s = \frac{M}{ad} = \frac{236.0}{1.44 \times 10.5} = 15.6 \text{ sq.in.}$$

The point of inflection from Table XIII is at a distance from the center of $0.28R_1$; therefore, with the bars arranged as in Fig. 36 the required length of bars is 2 X ($0.28R_1$ + 12 diameters) = 2 X (0.28 X 17.5 + 1.00) = 11.8 ft. By using ten l-in. round bars 12 ft. long (A_s = 15.8) the necessary area will be provided and there will be sufficient embedment beyond the edge of the capital.

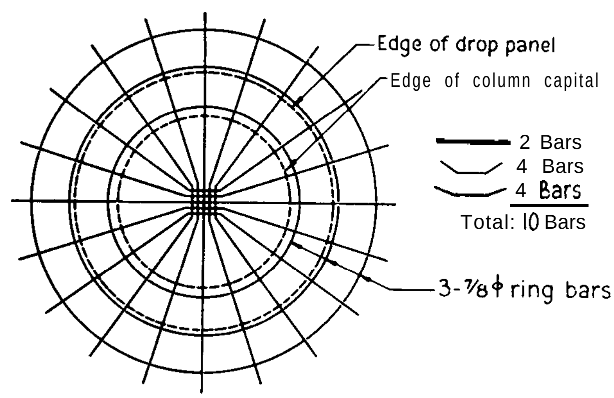

FIG. 36

Maximum negative tangential moment at columns. From Table XIII, at Point $0.20R_1$:M = $-0.0319pR_1^2$ = -0.0319 X 600 X 17.5^2 = -5,900 ft.lb. per ft. A_s = 5.9/1.44 X 7 = 0.59 sq.in. Use three ⅞-in. round bars spaced 12 in. o.c. Use an assembly of top bars as shown in Fig. 36, over each interior column.

*The value of R_1 may often be chosen as the distance between columns without appreciable loss in accuracy. If this is done, the calculations that follow will be simplified.

Maximum positive radial moment at mid-span. From Table XIII, coefficient per segment 1 ft. wide at a radius of 17.5 ft. = 0.0310 X 0.5 = 0.0155 at $0.5R_1$ and 0.0273 X 0.6 = 0.0164 at $0.6R_1$. Since the latter is greater, use moment on circumference of circle at $0.6R_1$ = $0.0273pR_1^2 \times 2\pi \times 0.6R_1$ = 0.0328π X 600 X 17.5^3 = 330,000 ft.lb. Divide this moment into six parts, one for each band connecting a column with each of the six adjacent columns. Note that the tank wall is taking the place of 12 columns in the regular three-way floor layout.

$$A_s = \frac{M/6}{ad} = \frac{330{,}000}{1.44 \times 7 \times 6} = 5.5 \text{ sq.in.}$$

Use seven l-in. round bars in each band. The point of inflection is 4.9 ft. from column, but it seems advisable to make these bars not less than 1 ft. longer than the minimum distance between adjacent edges of drop panels: 18.5 − 7.0 + 1.0 = 12.5 ft. There will be 12 such bands, six radiating from the center column and six between the intermediate columns.

Between the wall and the six intermediate columns there should be three positive steel bands radiating from each of six columns, a total of 18 bands. Use the same number and size of bars per band as between the columns, but fan them out as shown. Use twenty-one l-in. round bars radiating from each column as shown in Fig. 35*.

Maximum positive tangential moment at mid-span. These tangential moments are small as indicated by the maximum coefficient at Point $0.6R_1$ which is ½ (0.0099 + 0.0080) = 0.0090 (*see Table XIII*). The moment equals 0.0090 X pR_1^2 = 0.0090 X 600 X 17.5^2 = 1,700 ft.lb. per ft. A_s = 1.7/1.44 X 7 = 0.17. Use W-in. round bars spaced 12 in. o.c., five bars around each column with maximum radius of 18.5/2 = 9.25 ft., say 9 ft.

At the edge, where the slab and wall are continuous, use a moment-df $0.015p\,(D/2)2$ = 0.015 X 600 X 35^2 = 11,000 ft.lb. per ft.** A_s = 11.0/1.44 X 7 = 1.09 sq.in. Use l-in. round bars. Total circumference of tank wall = π X 70 = 220 ft. Number of bars = 1.09 X 220/0.79 = 304. Assuming the point of inflection at the quarter-point of the exterior span these radial bars should extend 0.25 X 16.5 = 4.13 ft. + 12 diameters = 5.13 ft., say 5 ft. 3 in. beyond the inside face of the wall. Tangential moment at a fixed edge equals radial moment times Poisson's ratio = 11,000 X 0.2 = 2,200 ft.lb. per ft. A_s = 2.2/1.44 X 6 = 0.25 sq.in. Use ⅝-in. round bars spaced 12 in. o.c., six rings required.

Section 16. General Procedure for Design of a lank

As mentioned in the "Introduction", to design a specific tank it is not necessary to make all the calculations in the preceding 12 sections. An illustration follows of a typical office procedure in designing a tank similar to that in Fig. 29 but with different dimensions.

Given:

H = 18 ft.
D = 46 ft.
w = 62.5 lb. per cu.ft.
$p = \begin{cases} 400 \text{ lb. per sq.ft. (4-ft. fill)} \\ 432 \text{ lb. per sq.ft. (surpressure)} \end{cases}$
$f_s = \begin{cases} 14{,}000 \text{ p.s.i. (for ring steel only)} \\ 20{,}000 \text{ p.s.i.} \end{cases}$
f_c = 300 p.s.i.
E_s = 30 X 10^6 p.s.i.
n = 10
c = 0.0003

Proceed as in Section 6 for an open-top tank.

Estimate t = 1.0 ft., then $H^2/Dt = \frac{18^2}{46 \times 1}$ = 7.04, say, 7

Multipliers of coefficients from
Table II, wHR = 62.5 X 18 X 23 = 25,900 lb. per ft.
Table IV, pR = 432 X 23 = 9,900 lb. per ft.

Check estimated thickness, t

Ring tension at $0.6H$
= 0.650 X 25,900 + 1.050 X 9,900
= 27,200 lb. per ft.

t = 0.0004 X 27,200 = 10.9 in. t = 1.0 ft. is ample. (*See Equation 2*).

Compute ring tension at intervals of $0.1H$ throughout height of wall (*Schedule A*).

Schedule A

Point	0.0H	0.1H	0.211	0.3H	0.4H	0.5H	0.611	0.7H	0.8H	0.9H	1.0H
Coef., Table II. Tri.		0.100	+ 0.216	+ 0.334							
Coef., Table IV. Rect.		1.000	+ 1.016	+ 1.0[illegible]	+11,700	+14,600	+16,800	+17,400	+15,100	+ 9,200	
Ring ten. Tri.	+0.[illegible]	[illegible]2,600	+10,10[illegible]5,600	+10,2[illegible]8,700	+10+10,5[illegible]0.565	+10,400[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Ring ten. Rect.		9,900	[illegible]0,100	[illegible]0,[illegible]							
Total ring ten.	+9,500	+12,500	+15,700	+18,900	+22,100	+25,100	+27,200	+27,000	+22,900	+13,700	0

Multipliers of coefficients from Table VIII
wH^3 = 62.5 X 18^3 = 364,500 ft.lb. per ft.
pH^2 = 432 X 18^2 = 140,000 ft.lb. per ft.

Compute moments at intervals of $0.1H$ throughout height of wall (*Schedule B*).

*A special investigation shows that the positive moment per ft. equals $0.0082p(D/2)^2$ at a Point $0.75D/2$ = 0.75 X 35 = 26.3 fc. from the center. Total moment = 0.0082 X 600 X 35^2 X 2π X 26.3 = 1,000,000 ft.lb. A_s = 1,000/1.44 X 7 = 99 sq.in. Thcrc arc 21 **bars** for each of six columns: A_s = 21 X 6 X 0.79 = 100 sq.in., which is sufficient.

**A thcorccically correct coefficient of 0.0169 was determined in the same way as those in Table XIII, but the 11 per cent reduction is considered permissible since the edge is not fully fixed.

Schedule B

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Coef., Table VIII	0	0	0	0	+0.0004	+0.0013	+0.0030	+0.0050	+0.0068	+0.0061	0
Mom. Tri.	0	0	0	0	+ 100	+ 500	+ 1,100	+ 1,800	+ 2,500	+ 2,200	0
Mom. Rect.	0	0	0	0	+ 100	+ 200	+ 400	+ 700	+ 1,000	+ 900	0
Total mom.	0	0	0	0	+ 200	+ 700	+ 1,500	+ 2,500	+ 3,500	+ 3,100	0

$V = 0.103\ (62.5 \times 18^2 + 432 \times 18) = 2{,}900$ lb. per ft.
(See Table XVI)

$$v = \frac{2{,}900}{0.875 \times 12 \times 10} = 28 \text{ p.s.i.}$$

Determine ring tension and moment in wall due to moment at top of wall induced by continuous top slab. As first step obtain fixed end moment at edge of slab as in Section 12.

Estimate c = 7 ft. $c/D = 7/46 = 0.152$, say, 0.15

Estimate top slab thickness = 10 in.

Fixed end moment including surpressure

$= -0.049\ (400 + 125 - 432) \times 23^2$

$= -2{,}400$ ft.lb. per ft.

Fixed end moment omitting surpressure

$= -0.049\ (400 + 125) \times 23^2$

$= -13{,}600$ ft.lb. per ft.

(See Table XIII for coefficient)

Distribution factors
for wall $= \dfrac{81}{81 + 14} = 0.85$
for slab $= \dfrac{14}{81 + 14} = 0.15$

Induced moment in wall, M
surpressure included $= 2{,}400 \times 0.85$
$= 2{,}000$ ft.lb. per ft.
surpressure omitted $= 13{,}600 \times 0.85$
$= 11{,}600$ ft.lb. per ft.

Multipliers of coefficients from

Table VI
$MR/H^2 = 2{,}000 \times 23/18^2$
$= 142$ lb. per ft.
$MR/H^2 = 11{,}600 \times 23/18^2$
$= 824$ lb. per ft.

Table XI
$M = 2{,}000$ ft.lb. per ft.
$M = 11{,}600$ ft.lb. per ft.

Compute ring tension and moment at intervals of 0.1 H *(Schedule C).*

Schedule C

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Coef., Table VI	0	+ 13.74	+ 14.80	+10.80	+ 6.21	+ 2.70	+ 0.60	− 0.36	− 0.66	− 0.70	− 0.64
Ring ten. Surpres. incl.	0	+ 2,000	+ 2,100	+1,500	+ 900	+ 400	+ 100	− 100	− 100	− 100	− 100
Ring ten. Surpres. omit.	0	+11,300	+12,200	+8,900	+5,100	+2,200	+ 500	− 300	− 500	− 600	− 500
Coef., Table XI	+ 1.000	+ 0.544	+ 0.215	+0.030	−0.050	−0.067	−0.051	−0.031	−0.014	−0.003	0
Mom. Surpres. incl.	+ 2,000	+ 1,100	+ 400	+ 100	− 100	− 100	− 100	− 100	0	0	0
Mom. Surpres. omit.	+11,600	+ 6,300	+ 2,500	+ 300	− 600	− 800	− 600	− 400	− 200	0	0

Combine ring tension and moment values in Schedules A, B, and C (Schedule D)

Schedule D

Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Ring ten. Tri., Schedule A	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	+14,600	+16,800	+17,400	+15,100	+ 9,200	[illegible]
Ring ten. Rec., Schedule A	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	+10,500	+10,400	+ 9,600	+ 7,800	+ 4,500	0
Ring ten. Sur. incl., Schedule C	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	+ 400	+ 100	− 100	− 100	− [illegible]	−100
Ring ten. Sur. omit., Schedule C	0	+11,300	112,200	+8,900	5,100	+ 2,200	+ [illegible]	[illegible]	[illegible]	[illegible]	−500
Max. ring ten.	+ 9,500	+14,500	+17,800	+20,400	+23,000	+25,500	+27,300	+26,900	+22,800	+13,600	−100
Mom. Tri., Schedule B	0	0	0	0	+ 100	+ 500	+ 1,100	+ 1,800	+ 2,500	+ 2,200	0
Mom. Rec., Schedule B	0	0	0	0	+ 100	+ 200	+ 400	+ 700	+ 1,000	+ 900	0
Mom. Sur. incl., Schedule C	+ 2,000	+ 1,100	+ 400	+ 100	− 100	− 100	− 100	− 100	0	0	0
Mom. Sur. omit., Schedule C	+11,000	+ 6,300	+ 2,500	+ 300	− 600	− 800	− 600	− 400	− 200	0	0
Max. mom.	+11,000	+ 6,300	+ 2,500	+ 300	+ 100 − 600	+ 600 − 800	+ 1,400 − 600	+ 2,400 − 400	+ 3,500 − 200	+ 3,100 0	0

Stiffness of wall (Table XVIII for $H^2/Dt = 7$)
$= 0.843 \times 12^3/18 = 81$

Stiffness of slab (Table XIX for $c/D = 0.15$)
$= 0.332 \times 10^3/23 = 14$

Maximum area of ring steel is

$$A_s = \frac{27{,}300}{14{,}000} = 1.95 \text{ sq.in. per ft}$$

Use ¾-in. round bars spaced 5½ in. o.c. in each of two curtains from the $0.6H$ point to bottom of wall (see Section 8). Above $0.6H$ reduce ring steel in proportion to the ring tension values in Schedule D. Check stress in concrete by Equation 1:

$$f_c = \frac{0.0003 \times 30 \times 10^6 \times 1.92 + 27{,}300}{12 \times 12 + 10 \times 1.92} = 273 \text{ p.s.i.}$$

$$v = \frac{574{,}400}{0.875 \times 2\pi \times 23 \times 12 \times 8.5} = 45 \text{ p.s.i.}$$

Compute radial moments in slab by multiplying coefficients from Tables XIII and XV by $pR^2 = 525 \times 23^2 = 278{,}000$ ft.lb. per ft. and $M = 13{,}600 - 11{,}600 = 2{,}000$ ft.lb. per ft., respectively (*Schedule E*).

Schedule E

Point	0.15R	0.2R	0.25R	0.3R	0.4R	0.5R	0.6R	0.7R	0.8R	0.9R	1.0R
Coef., Table XIII. Fixed	-0.1089	-0.0521	-0.0200	+0.0002	+0.0220	+0.0293	+0.0269	+0.0169	+0.0006	-0.0216	-0.0490
Coef., Table XV. Mat edge	-1.594	-0.930	-0.545	-0.280	+0.078	+0.323	+0.510	+0.663	+0.790	+0.900	+1.000
Rad. mom. Fixed	-30,300	-14,500	-5,600	+100	+6,100	+8,100	+7,500	+4,700	+200	-6,000	-13,600
Rad. mom. Mat edge	-3,200	-1,900	-1,100	-600	+200	+600	+1,000	+1,300	+1,600	+1,800	+2,000
Total rad. mom., per ft.	-33,500	-16,400	-6,700	-500	+6,300	+8,700	+8,500	+6,000	+1,800	-4,200	-11,600
Total rad. mom.. per seg.	-5,000	-3,300	-1,700	-200	+2,500	+4,400	+5,100	+4,200	+1,400	-3,800	-11,600

Maximum area of vertical steel is

$$A_s = \frac{11.0}{1.44 \times 10} = 0.76 \text{ sq.in. per ft}$$

Use ⅝-in. round bars spaced 5 in. o.c. at top of wall in outside curtain. Provide reinforcement elsewhere in proportion to the moment values in Schedule D but not less than ½-in. round bars spaced 18 in. o.c. in inside and outside curtains.

Continue with design **of** *top slab as in Section 12.*

Column load

$= 1.007 \times 525 \times 23^2 + 9.29\,(13{,}600 - 11{,}600)$

$= 298{,}600$ lb. *(See Table XVII)*

From *Reinforced Concrete Design Handbook* (Table 18):

Load on concrete in 22-in. round tied column $= 380 \times 540 = 205{,}000$ lb.

Load on reinforcement $= 298{,}600 - 205{,}000 = 93{,}600$ lb.

Use ten 1-in. round bars.

Radius of critical section for shear around column capital considering 5-in. drop panel:

$(42 + 15 - 1.5) = 55.5$ in. $= 4.62$ ft.

$$v = \frac{V}{0.875bd} = \frac{298{,}600 - 525 \times \pi \times 4.62^2}{0.875 \times 2\pi \times 55.5 \times 13.5} = 64 \text{ p.s.i.}$$

Radius of critical section for shear around 10.5-ft. diameter drop panel is

$(63 + 10 - 1.5) = 71.5$ in. $= 5.96$ ft.

$$v = \frac{298{,}600 - 525 \times \pi \times 5.96^2}{0.875 \times 2\pi \times 71.5 \times 8.5} = 72 \text{ p.s.i.}$$

Shear at edge of wall $= \pi \times 23^2 \times 525 - 298{,}600 = 574{,}400$ lb.

Negative radial moment at edge of column capital $= -33{,}500 \times 7\pi\,(1 - 0.28) = -530{,}000$ ft.lb. *(see Section 12).*

$$A_s = \frac{530}{1.44 \times 13.5} = 27.3 \text{ sq.in.}$$

Use eighteen 1¼-in. square bars in top of slab arranged as in Fig. 31.

Maximum positive radial moment at $0.6R$

$= 8{,}500 \times 2\pi \times 0.6 \times 23 = 736{,}000$ ft.lb.

$$A_s = \frac{736}{1.44 \times 8.5} = 60.0 \text{ sq.in.}$$ Use one hundred thirty-eight ¾-in. round bars.

$$\text{Spacing at } 0.6R = \frac{2\pi \times 0.6 \times 23 \times 12}{138} = 7\tfrac{1}{2} \text{ in}$$

Maximum negative moment at inside of wall

$= -11{,}600 \times 2\pi \times 23 = 1{,}675{,}000$ ft.lb.

$$A_s = \frac{1{,}675}{1.44 \times 8.5} = 137 \text{ sq.in.}$$ Use two hundred twenty-eight ⅞-in. round bars.

$$\text{Spacing at wall} = \frac{2\pi \times 23 \times 12}{228} = 7\tfrac{1}{2} \text{ in.}$$

Determine the proper length of bars by sketching the moment curve as described in Section 12.

Compute tangential moments in slab by multiplying coefficients from Tables XIII and XV by $pR^2 = 278{,}000$ ft.lb. per ft. and $M = 2{,}000$ ft.lb., respectively *(Schedule F).*

Schedule F

Point	0.15R	0.2R	0.25R	0.3R	0.4R	0.5R	0.6R	0.7R	0.8R	0.9R	1.0R
Coef., Table XIII. Fixed	-0.0218	-0.0284	-0.0243	-0.0177	-0.0051	+0.0031	+0.0080	+0.0086	+0.0057	-0.006	-0.0098
Coef., Table XV. M at edge	-0.319	-0.472	-0.463	-0.404	-0.251	-0.100	+0.035	+0.157	+0.263	+0.363	+0.451
Tan. mom. Fixed	-6,100	-7,900	-6,800	-4,900	-1,400	+900	+2,200	+2,400	+1,600	-200	-2,700
Tan. mom. M at edge	-600	-900	-900	-800	-500	-200	+100	+300	+500	+700	+900
Total tan. mom.. per ft.	-6,700	-8,800	-7,700	-5,700	-1,900	+700	+2,300	+2,700	+2.100	+500	-1,800

Maximum steel area will be required at edge of drop panel.

$$A_s = \frac{8.3}{1.44 \times 8.5} = 0.68 \text{ sq.in. per ft.}$$ Use 1-in. round bars at 14 in. o.c.

Steel required for positive moment at $0.7R$:

$$A_s = \frac{2.7}{1.44 \times 8.5} = 0.22 \text{ sq.in. per ft.}$$ Use ½-in. round bars at 11 in. o.c.

Determine the number of bars required for tangential moment by sketching the moment diagram.

Section 17. Effect of Variation in Wall Thickness

All tables and numerical examples in preceding sections are based on the assumption that the wall has uniform thickness from top to base. The effect of tapering the wall will now be discussed.

A wall with two curtains of reinforcement should preferably be not less than 8 in. In the examples in the preceding sections, 15 in. is the thickness required for maximum ring tension which occurs at a depth of approximately $0.6H$ below the top. Actually, only the upper one-half of the wall can be tapered and the thickness reduced from 15 to 8 in. The cross-sectional area of the wall can then be reduced from 1.25 X 20 = 25.0 sq.ft. to 25.0 − 0.5 X 0.58 X 10 = 22.1 sq.ft. The reduction is hardly sufficient to offset the added cost of forms for the tapered circular wall.

Gray* has presented data for wall sections that vary from a maximum at the base to zero at the top. For illustration, consider a wall with $H = 20$ ft., $D = 54$ ft., and $t = 1.25$. For this wall, Gray's data show that maximum ring tension is approximately 8 per cent greater for triangular than for rectangular wall section, that is, when the sectional area is reduced from 25.0 to 12.5 sq.ft. For the reduction of 2.9 sq.ft. in the foregoing paragraph it may be estimated roughly that the increase in maximum ring tension will be 2.9 X 8/12.5 = 2 per cent. At any rate, the increase appears to be negligible.

Timoshenko** gives an example with $H = 14$ ft. and $D = 60$ ft. The wall thickness is 14 in. in one case but varies from 14 to 3.5 in. in the other case. Moment and shear at the base are as follows:

	Moment, in.lb.	Shear, lb.
Uniform thickness, 14 in. :	13,960	564
Variable thickness (3.5 to 14 in.):	13,900	527

It is seen that the moment is practically unchanged and the shear is reduced by only 6.5 per cent. The change will be even smaller when the taper is from 14 in. at mid-height to 8 in, at top. In this case, the taper may be ignored, but under extreme circumstances it may be advisable to take it into account. This may be done approximately by inserting in H^2/Dt the value oft which exists at the point being investigated in the wall, or, in other words, to use values of H^2/Dt which vary from top to base.

Section 18. Temperature Stresses in Cylindrical lank Walls

Tanks containing hot liquids are subject to temperature stresses. Assume that the temperature is T_1 in the inner face, T_2 in the outer face, and that the temperature decreases uniformly from inner to outer face, $T_1 - T_2$ being denoted as T. Fig. 37 shows a segment of a tank wall in two positions, one before and one after a uniform increase in temperature. The original length of the arc of the wall has been increased, but an increase that is uniform throughout will not create any stresses as long as the ring is supposed to be free and unrestrained at its edges. It is the temperature differential only, T, which creates stresses.

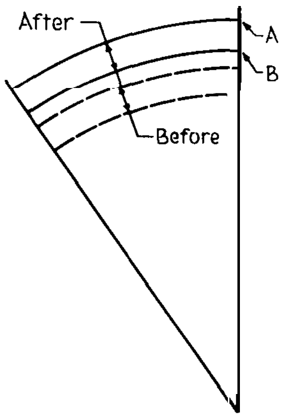

FIG. 37

The inner fibers being hotter tend to expand more than the outer fibers, so if the segment is cut loose from the adjacent portions of the wall, Point A in Fig. 38 will move to A', B will move to B', and section

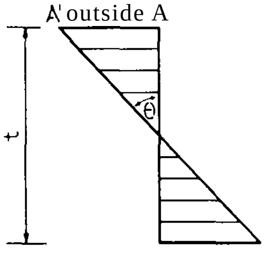

FIG. 38

AB, which represents the stressless condition due to a uniform temperature change throughout, will move to a new position $A'B'$. Actually the movements from A to A' and B to B' are prevented since the circle must remain a circle, and stresses will be created that are proportional to the horizontal distances between AB and $A'B'$.

*See Bibliography, reference No. 18.
**See Bibliography, reference No. 17

It is clear that AA' = BB' = movement due to a temperature change of $\frac{1}{2}T$ or when e is the coefficient of expansion, that

$AA' = BB' = \frac{1}{2}T \times e$ per unit length of arc,

and

$$\theta = \frac{AA'}{\frac{1}{2}t} = \frac{T \times e}{t}.$$

In a homogeneous section, the moment M required to produce an angle change θ in an element of unit length may be written as

$$M = EI\theta$$

Eliminating θ gives

$$M = \frac{EI \times T \times e}{t}$$

The stresses in the extreme fibers created by M are

$$f = \frac{M}{I} \times \frac{t}{2} = \frac{1}{2}E \times T \times e$$

The stress distribution across the cross section is as indicated in Fig. 38. The stresses are numerically equal at the two faces but have opposite signs. Note that the equation applies to uncracked sections only, and that this procedure of stress calculation is to be considered merely as a method by which the problem can be approached. The variables E and I in the equations are uncertain quantities. E may vary from 1,500,000 up to 4,500,000 p.s.i., and Z may also vary considerably because of deviations from the assumption of linear relation between stress and strain. Finally, if the concrete cracks, M can no longer be set equal to $EI\theta$, nor f equal to $\frac{M}{I} \times \frac{t}{2}$. As a result, the equation $f = \frac{1}{2}ETe$ is to be regarded as merely indicative rather than formally correct.

The value of e may be taken as 0.000006, and for the purpose of this problem choose $E = 1{,}500{,}000$ p.s.i. Then $E \times e = 9$ and $f = 4.5T$.

The value of T is the difference between temperatures in the two surfaces of the concrete which may be computed from the temperature of the stored liquid and the outside air.

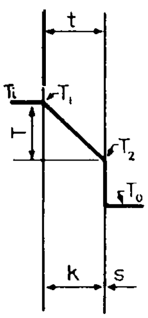

FIG. 39

When the flow of heat is uniform from the inside to the outside of the wall section in Fig. 39, the temperature difference, $T = T_1 - T_2$, is smaller than the difference, $T_i - T_o$, between the inside liquid and the outside air. Standard textbooks give

$$T = (T_i - T_o)\,\frac{t/k}{1/k}$$

in which

$$\frac{1}{k} = \frac{1}{s} + \frac{t}{k} + \frac{t_1}{k_1}$$

k = coefficient of conductivity of stone or gravel concrete = 12 B.t.u. per hour per sq.ft. per deg. F. per in. of thickness

k_1 = coefficient of conductivity of insulating material

t = thickness of concrete in in.

t_1 = thickness of insulating material

s = outside surface coefficient = 6 B.t.u. per hour per sq.ft. per deg. F.

Assuming an uninsulated wall

$$T = (T_i - T_o)\,\frac{\frac{t}{12}}{\frac{1}{6} + \frac{t}{12}} = (T_i - T_o)\,\frac{t}{2 + t}$$

Consider a tank with wall thickness t = 10 in. which holds a liquid with a temperature T_i = 120 deg. F. while the temperature of the outside air T_o = 30 deg. F. Then

$$T = (120 - 30) \times \frac{10}{2 + 10} = 75 \text{ deg. F.}$$

and

$$f = 4.5T = 4.5 \times 75 = 375 \text{ p.s.i}$$

The stress off = 375 p.s.i. is tension in the outside and compression in the inside face. If the uniformly distributed ring tension due to load in the tank is, say, 300 p.s.i., the combined stress will be:

Outside fiber: $300 + 375 = 675$ p.s.i. (tension)
Inside fiber: $300 - 375 = -75$ p.s.i. (compression)

In reality, too much significance should not be attached to the temperature stress computed from the equation derived. The stress equation is developed from the strain equation, $AA' = \frac{1}{2}Te$, based on the assumption that stress is proportional to strain. This assumption is rather inaccurate for the case under discussion. The inaccuracy may be rectified to some extent by using a relatively low value for E_c, such as $E_c = 1{,}500{,}000$ p.s.i., which is used in this section. An even lower value may be justified.

As computed in the example, a temperature differential of 75 deg. F. gives a stress of 375 p.s.i. in the extreme fiber. This is probably more than the concrete can take in addition to the regular ring tension stress without cracking on the colder surface. The temperature stress may be reduced by means of insulation, which serves to decrease the temperature differential, or additional horizontal reinforcement may be provided close to the colder surface. A procedure will be illustrated for determination of temperature steel. It is not based upon a rigorous mathematical analysis but will be helpful as a guide and as an aid to engineering judgment.

It is proposed to base the design on the moment derived in this section, $M = E_c I T e/t$, in which the value of E_c is taken as 1,500,000 p.s.i. If I is taken for a section 1 ft. high, I equals t^3, and M is the moment per ft. Then

$M = 1{,}500{,}000 \times t^2 \times T \times 0.000006 = 9t^2T$ in.lb. per ft. in which

t = thickness of wall in in.
T = temperature differential in deg. F.

The area of horizontal steel at the colder face computed as for a cracked section is

$$A_s = \frac{M}{\frac{7}{8} f_s d} = \frac{9t^2T}{17{,}500d} \text{ sq.in. per ft.}$$

For example, assume $t = 15$ in., $T = 75$ deg. F., and $d = 13$ in., which gives

$$A_s = \frac{9 \times 15^2 \times 75}{17{,}500 \times 13} = 0.67 \text{ sq.in. per ft.}$$

This area is in addition to the regular ring steel.

Section 19. Details

Reference has been made in previous sections to sliding, hinged, or continuous joints at base of wall. A detail for each of these three types of joints is shown in Fig. 40. For a sliding base, the bearing surface on the wall, to calk part of the groove with oakum and to fill the remainder of it with mastic. If excellent workmanship and materials are employed in calking the groove, the dam may be omitted. By mastic is meant tar, asphalt or synthetic material placed in accordance with manufacturer's direction and known to be resistant to the liquid stored.

In the continuous base, vertical reinforcement extends across the joint which is to be prepared so as to develop maximum bond. Good bond qualities are obtained by the following procedure. After concrete is placed, but just before initial set-about six hours—clean the joint surface with a pressure water jet. Then cover the joint and keep it continually wet. Just before new concrete is placed, flush the old surface with 1:2 portland cement mortar. Vibrate the new concrete and keep it moist for several days. This procedure may give as high as 96 per cent efficiency and only little loss in efficiency occurs in delaying the placing of new concrete up to 20 days.

To make certain that the continuous base is watertight, a dam may be placed as shown. The position of the joint which in Fig. 40 is a few inches above the top of the footing facilitates placing of the wall forms, but this is not essential.

It may sometimes be desirable to avoid transmitting moment from base slab into the wall, and the hinged joint detail in Fig. 40 may then be used. The reinforcing bars are crossed at the center of the joint, and grooves at the outside edges of the joint are calked with oakum. This joint can transmit little moment but may have to carry horizontal shear, so the middle

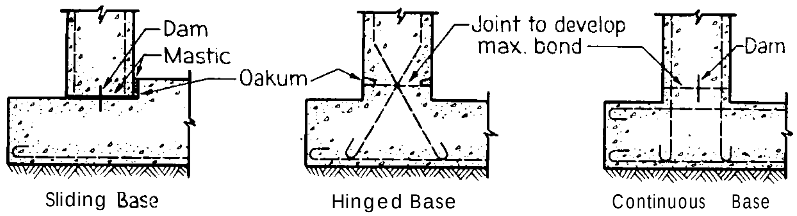

FIG. 40

top of the footing is given a trowel finish and then covered with mastic. To insure watertightness in the joint a dam is placed midway between the two wall surfaces. Here and in the following, a dam means a waterstop made of sheet copper, galvanized iron, rubber, soft wood, synthetic material, or fabric impregnated with asphalt or pitch. Whatever material is used, it should be resistant to attack by the liquid stored and must not be ruptured by the small movements that may occur in the joint.

Another means of making a sliding joint watertight is to provide a groove as shown at the inside of portion of the bearing surface should be prepared as described above to develop maximum bond. It is not considered necessary to use the hinged joint when the wall is supported on an ordinary wall footing since such a footing can transmit little moment to the subgrade.

A base slab on fill is generally divided by means of joints into a number of approximately equal areas. A common arrangement of such joints is shown in Fig. 41 together with the order of placing the concrete. The idea is to reduce the effect of shrinkage as much as possible. All the joints in the base slab must be made watertight.

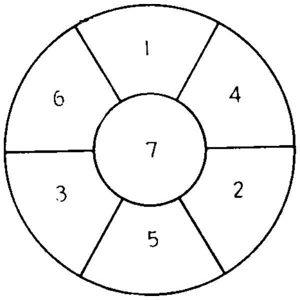

FIG. 41

At the wall footing in Fig. 42, the slab is resting on a shelf with troweled finish covered with mastic. The slab and the footing are made flush on top in order to keep the wall as low as possible, to avoid waste of concrete in the floor slab and to reduce the length of joint. At the interior column in Fig. 42, the slab is

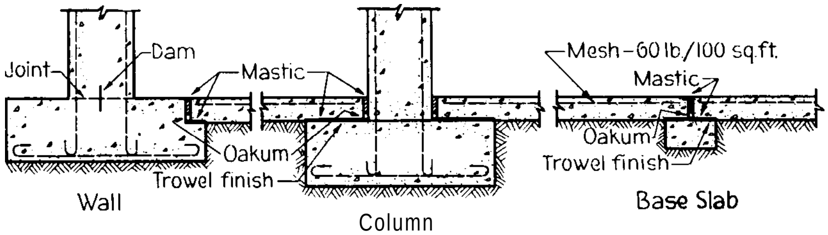

FIG. 42

placed on top of the footing to make the joint as short as possible. A continuous concrete sill block is indicated for all intermediate joints. The joints in Fig. 42 are calked with oakum and then filled with mastic. A 60-lb. mesh is placed near the top of the slab.

Two types of joints at top of wall are shown in Fig. 43. The wall joint with free or sliding top has

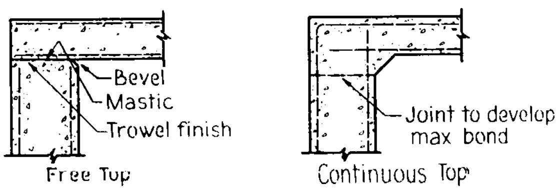

FIG. 43

troweled surface covered with mastic. The inside corner of the wall is beveled to minimize the danger of spalling. The continuous joint is designed to carry the calculated moment, and the joint surface is treated to develop maximum bond. A fillet as shown is desirable but not essential. Whether the free or the continuous joint should be used depends upon circumstances. A continuous joint will be more airtight, and the edge moment it can develop will often be useful for the design of the roof slab.

The best practice is to use as few joints as feasible in the wall, since any joint is a potential source of leakage unless it is given careful attention both in design and construction. If built right, intermediate joints in the wall are not objectionable, however, and they may have to be employed if required by limitations in the working capacity of the mixing and placing facilities available.

In general, the vertical joint is preferred to a horizontal joint because it requires less attention to make it watertight. The vertical joint shown in Fig. 44 has a dam in the middle of the wall, and the outside corners are beveled so as to conceal the crack that may occur in the joint. The notch shown in the inside of the joint is optional but appears to be desirable. In case the dam should deteriorate to the point where the joint leaks, the notch can easily be calked and the joint made watertight again.

A joint as that in Fig. 44 is built by means of a vertical board or bulkhead which must be notched for passage of the ring bars. After the board is removed, the new concrete is simply cast against the old concrete, and as a result the adhesion or bonding qualities may not be sufficiently high to prevent the joint from cracking when the wall is subjected to ring tension.

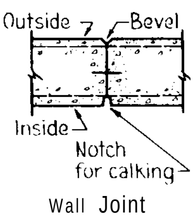

FIG. 44

In contrast to the joint discussed, a horizontal joint can be made highly resistant to tensile forces. The reason it has been avoided is probably that special attention is required to make it watertight. However, if the procedure for developing maximum bond described above is specified, and the specification is enforced on the job, the type of continuous joint shown in Fig. 40 will give good performance not only at the base but anywhere in the wall. A dam should be specified for all horizontal joints.

Concrete in circular tank walls should be placed in horizontal lifts of not over 2 ft. The concrete should be deposited at frequent intervals around the periphery of the tank. No temporary joints should be allowed to become "cold" before the adjacent concrete is placed. The time interval varies, but usually it should not be more than 45 minutes. If necessary, two or more placing crews may be employed in order to adhere to this 45-minute limitation, or the height of the lift may be reduced. Also, it is advisable to safeguard against complete stoppage of concrete placing by having two mixers available.

Curing is of prime importance, especially for the tank walls. It reduces the shrinkage stresses, increases the concrete strength and improves the watertightness. Specifications should be explicit in demanding the best possible kind of curing that can be obtained at reasonable cost with the facilities available at the job site. A continual spray of water from perforated pipes suspended around the rim of the tank wall makes for excellent curing, and so does a complete curtain of hurlap kept soaking wet by spraying from a hose.

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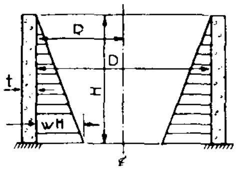
C

Table I

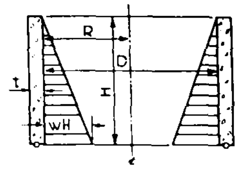
Tension in circular rings
Triangular load
Fixed base, free top
T = coef. X wHR lb. per ft.
Positive sign indicates tension



H^2/Dt	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	10.149	+0.134	+0.120	to.101	10.082	+0.066	+0.049	to.029	+0.014	10.00"
0.8	+0.263	+0.239	to.215	10.190	+0.160	10.130	+0.096	+0.063	+0.034	+0.010
1.2	+0.283	+0.271	+0.254	+0.234	+0.209	+0.180	+0.142	+0.099	+0.054	+0.016
1.6	+0.265	+0.268	+0.268	+0.266	+0.250	+0.226	+0.185	to.134	+0.075	+0.023
2.0	+0.234	+0.251	+0.273	+0.285	+0.285	10.274	+0.232	+0.172	10.104	+0.031
3.0	+0.134	+0.203	+0.267	to.322	+0.357	+0.362	10.330	+0.262	+0.157	+0.052
4.0	+0.067	+0.164	+0.256	+0.339	to.403	+0.429	+0.409	10.334	+0.210	+0.073
5.0	+0.025	+0.137	+0.245	+0.346	+0.428	+0.477	+0.469	t0.398	+0.259	10.092
6.0	+0.018	+0.119	+0.234	+0.344	+0.441	+0.504	+0.514	to.447	to.301	+0.112
8.0	-0.011	+0.104	to.219	to.335	to.443	10.534	+0.575	to.530	+0.381	10.151
10.0	-0.011	+0.098	+0.208	+0.323	to.437	+0.542	+0.608	10.589	+0.440	+0.179
12.0	-0.005	+0.097	10.202	+0.312	+0.429	+0.543	+0.628	to.633	+0.494	10.211
14.0	-0.002	+0.098	+0.200	+0.306	to.420	+0.539	+0.639	10.666	+0.541	+0.241
16.0	0.000	+0.099	+0.199	10.304	to.412	+0.531	+0.641	+0.687	+0.582	+0.265

Table II

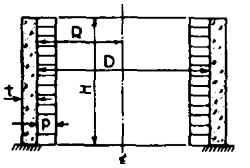
Tension in circular rings
Triangular load
Hinged base, free top
T = coef. X wHR lb. per ft.
Positive sign Indicates tension



H^2/Dt	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.611	0.711	0.8H	0.9H
0.4	+0.474	to.440	+0.395	to.352	+0.308	+0.264	+0.215	+0.165	+0.111	+0.057
0.8	+0.423	10.402	10.381	10.358	10.330	+0.297	+0.249	10.202	+0.145	+0.076
1.2	+0.350	+0.355	+0.361	+0.362	10.358	+0.343	+0.309	+0.256	+0.186	+0.098
1.6	+0.271	+0.303	+0.341	+0.369	+0.385	+0.385	+0.362	to.314	+0.233	+0.124
2.0	+0.205	+0.260	to.321	to.373	+0.411	+0.434	+0.419	+0.369	+0.280	+0.151
3.0	+0.074	+0.179	+0.281	10.375	+0.449	+0.506	+0.519	10.479	10.375	to.210
4.0	10.017	+0.137	+0.253	+0.367	+0.469	+0.545	+0.579	to.553	+0.447	+0.256
5.0	-0.008	+0.114	+0.235	+0.356	+0.469	+0.562	+0.617	+0.606	+0.503	+0.294
6.0	-0.011	+0.103	+0.223	+0.343	+0.463	+0.566	+0.639	+0.643	10.547	+0.327
8.0	-0.015	+0.096	+0.208	+0.324	10.443	+0.564	10.661	+0.697	+0.621	+0.386
10.0	-0.008	to.095	+0.200	+0.311	+0.428	+0.552	+0.666	to.730	+0.678	to.433
12.0	-0.002	+0.097	+0.197	+0.302	10.417	+0.541	+0.664	+0.750	+0.720	+0.477
14.0	0.000	+0.098	+0.197	+0.299	+0.408	+0.531	+0.659	+0.761	+0.752	+0.513
16.0	+0.002	+0.100	+0.198	+0.299	+0.403	to.521	+0.650	+0.764	+0.776	+0.536

Table III

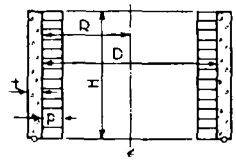
Tension in circular rings
Rectangular load
Fixed base, free top
T = coef. X pR lb. per ft.
Positive sign indicates tension



H^2/Dt	0.0H	0.1H	0.2H	0.311	0.4H	0.511	0.6H	0.711	0.8H	0.9H
0.4	+0.582	+0.505	+0.431	+0.353	+0.277	to.206	to.145	to.092	+0.046	+0.013
0.8	t1.052	+0.921	+0.796	+0.669	+0.542	+0.415	to.289	to.179	+0.089	+0.024
1.2	+1.218	+1.078	+0.946	+0.808	+0.665	to.519	+0.378	+0.246	to.127	+0.034
1.6	+1.257	+1.141	+1.009	+0.881	+0.742	+0.600	to.449	to.294	to.153	+0.045
2.0	+1.253	+1.144	+1.041	+0.929	+0.806	+0.667	to.514	to.345	+0.186	+0.055
3.0	+1.160	+1.112	+1.061	+0.998	+0.912	+0.796	r-0.646	to.459	to.258	+0.081
4.0	+1.085	+1.073	+1.057	+1.029	+0.977	+0.887	+0.746	to.553	+0.322	+0.105
5.0	al.037	+1.044	+1.047	+1.042	t1.015	to.949	+0.825	+0.629	to.379	+0.128
6.0	+1.010	+1.024	+1.038	+1.045	t1.034	+0.986	+0.879	+0.694	to.430	+0.149
8.0	+0.989	+1.005	+1.022	+1.036	+1.044	+1.026	+0.953	+0.788	+0.519	+0.189
10.0	+0.989	+0.998	+1.010	+1.023	t1.039	+1.040	to.996	+0.859	+0.591	+0.226
12.0	+0.994	+0.997	11.003	+1.014	cl.031	cl.043	+1.022	to.911	+0.652	+0.262
14.0	+0.997	+0.998	+ 1.000	11.007	+1.022	+1.040	+1.035	to.949	to.705	+0.294
16.0	+1.000	+0.999	+0.999	il.003	11.015	+1.032	11.040	+0.975	+0.750	+0.321

Table IV

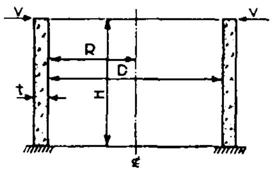
Tension in circular rings
Rectangular load
Hinged base, free top
T = coef. X pR lb. per ft.
Positive sign indicates tension



H^2/Dt	0.0H	0.1H	0.2H	0.311	0.4H	0.511	0.6H	0.7H	0.811	0.9H
0.4	+1.474	+1.340	+1.195	+1.052	+0.908	+0.764	10.615	+0.465	to.311	+0.154
0.8	+1.423	t1.302	+1.181	+1.058	+0.930	to.797	+0.649	+0.502	co.345	+0.166
1.2	+1.350	t1.255	+1.161	+1.062	+0.958	+0.843	to.709	t0.556	+0.386	+0.198
1.6	+1.271	+1.203	+1.141	cl.069	+0.985	+0.885	+0.756	+0.614	co.433	+0.224
2.0	+1.205	t1.160	+1.121	r1.073	+1.011	to.934	+0.819	+0.669	+0.480	+0.251
3.0	+1.074	t1.079	+1.081	11.075	+1.049	11.006	to.919	+0.779	+0.575	+0.310
4.0	+1.017	t1.037	+1.053	+1.067	+1.069	+1.045	to.979	to.853	to.647	+0.356
5.0	+0.992	t1.014	+1.035	+1.056	+1.069	+1.062	+1.017	+0.906	+0.703	+0.394
6.0	+0.989	ti.003	+1.023	tl.043	+1.063	cl.066	11.039	to.943	to.747	+0.427
8.0	+0.985	+0.996	+1.008	+1.024	+1.043	t1.064	+1.061	10.997	+0.821	+0.486
10.0	+0.992	to.995	+1.000	+1.011	+1.028	+1.052	+1.066	+1.030	+0.878	+0.533
12.0	+0.998	+0.997	+0.997	+1.002	+1.017	+1.041	+1.064	cl.050	+0.920	+0.577
14.0	+1.000	to.998	+0.997	to.999	+1.008	+1.031	11.059	+1.061	to.952	+0.613
16.0	+1.002	~1.000	+0.998	to.999	+1.003	+1.021	11.050	cl.064	+0.976	+0.636

Table V

Tension in circular rings
Shear per ft., V, applied at top
Fixed base, free top
T = coef. X VR/H lb. per ft.
Positive sign indicates tension

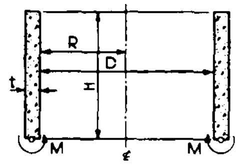


H^2/Dt	0.0H	0.1H	0.2H	0.3H	0.4H	0.511	0.6H	0.711	0.8H	0.9H
0.4	1.57	- 1.32	- 1.08	- 0.86	- 0.65	- 0.47	- 0.31	- 0.18	- 0.08	- 0.02
0.8	- 3.09	- 2.55	- 2.04	- 1.57	- 1.15	- 0.80	- 0.51	- 0.28	- 0.13	- 0.03
1.2	3.95	- 3.17	- 2.44	- 1.79	- 1.25	- 0.81	- 0.48	- 0.25	- 0.10	- 0.02
1.6	- 4.57	- 3.54	- 2.60	- 1.80	- 1.17	- 0.69	- 0.36	- 0.16	- 0.05	- 0.01
2.0	- 5.12	- 3.83	- 2.68	- 1.74	- 1.02	- 0.52	- 0.21	- 0.05	+ 0.01	+ 0.01
3.0	6.32	4.37	- 2.70	- 1.43	- 0.58	- 0.02	+ 0.15	+ 0.19	+ 0.13	+ 0.04
4.0	- 7.34	- 4.73	- 2.60	- 1.10	- 0.19	+ 0.26	+ 0.38	+ 0.33	+ 0.19	+ 0.06
5.0	- 8.22	- 4.99	- 2.45	- 0.79	+ 0.11	+ 0.47	+ 0.50	+ 0.37	+ 0.20	+ 0.06
6.0	9.02	5.17	- 2.27	- 0.50	+ 0.34	+ 0.59	+ 0.53	+ 0.35	+ 0.17	+ 0.01
8.0	-10.42	5.36	- 1.85	- 0.02	+ 0.63	+ 0.66	+ 0.46	+ 0.24	+ 0.09	+ 0.01
10.0	-11.67	- 5.43	- 1.43	+ 0.36	+ 0.78	+ 0.62	+ 0.33	+ 0.12	+ 0.02	0.00
12.0	-12.76	- 5.41	- 1.03	+ 0.63	+ 0.83	+ 0.52	+ 0.21	+ 0.04	- 0.02	0.00
14.0	-13.77	- 5.34	- 0.68	+ 0.80	+ 0.81	+ 0.42	+ 0.13	0.00	- 0.03	- 0.01
16.0	-14.74	- 5.22	- 0.38	+ 0.96	+ 0.76	+ 0.32	+ 0.05	0.04	- 0.05	0.02

*When this table is used for shear applied at the base. whilethetop is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Table VI

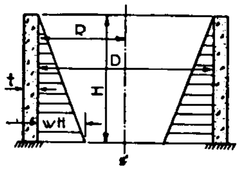
Tension in circular rings
Moment per ft., **M**, applied at base
Hinged base, free top
T = coef. X MR/H^2 lb. per ft.
Positive sign indicates tension



H^2/Dt	0.0H	0.1H	0.211	0.3H	0.4H	0.511	0.611	0.7H	0.811	0.911
0.4	+ 2.70	+ 2.50	+ 2.30	+ 2.12	+ 1.91	+ 1.69	+ 1.41	+ 1.13	+ 0.80	+ 0.44
0.8	+ 2.02	+ 2.06	+ 2.10	+ 2.14	+ 2.10	+ 2.02	+ 1.95	+ 1.75	+ 1.39	+ 0.80
1.2	+ 1.06	+ 1.42	+ 1.79	+ 2.03	+ 2.46	+ 2.65	+ 2.80	+ 2.60	+ 2.22	+ 1.37
1.6	+ 0.12	+ 0.79	+ 1.43	+ 2.04	+ 2.72	+ 3.25	+ 3.56	+ 3.59	+ 3.13	+ 2.01
2.0	- 0.68	+ 0.22	+ 1.10	+ 2.02	+ 2.90	+ 3.69	+ 4.30	+ 4.54	+ 4.08	+ 2.75
3.0	- 1.78	- 0.71	+ 0.43	+ 1.60	+ 2.95	+ 4.29	+ 5.66	+ 6.58	+ 6.55	+ 4.73
4.0	- 1.87	- 1.00	- 0.08	+ 1.04	+ 2.47	+ 4.31	+ 6.34	+ 8.19	+ 8.82	+ 6.81
5.0	- 1.54	- 1.03	- 0.42	+ 0.45	+ 1.86	+ 3.93	+ 6.60	+ 9.41	+11.03	+ 9.02
6.0	- 1.04	- 0.86	- 0.59	- 0.05	+ 1.21	+ 3.34	+ 6.54	+10.28	+13.08	+11.41
8.0	- 0.24	- 0.53	- 0.73	- 0.67	- 0.02	2.05	+ 5.87	+11.32	+16.52	+16.06
0.0	+ 0.21	- 0.23	- 0.64	- 0.94	- 0.73	+ 0.82	+ 4.79	+11.63	+19.48	+20.87
2.0	+ 0.32	- 0.05	- 0.46	- 0.96	- 1.15	- 0.18	+ 3.52	+11.27	+21.80	+25.73
4.0	+ 0.26	+ 0.04,	- 0.28	0.76	- 1.29	- 0.87	+ 2.29	+10.55	+23.50	+30.34
6.0	+ 0.22	+ 0.07	- 0.08	0.64	- 1.28	- 1.30	+ 1.12	+ 9.67	+24.53	+34.65

*When this table is used for moment applied at the top, while the top is hinged, 0.0H is the bottom of the wall and 1.011 is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Table VII

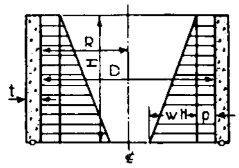


Moments in cylindrical wall
Triangular load
Fixed base, free top

Mom. = coef. X wH^3 ft.lb. per ft.
Positive sign indicates tension in the outside

$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point									
0.4	+.0005	+.0014	+.0021	+.0007	-.0042	-.0150	-.0302	-.0529	-.0816	-.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	-.0068	-.0224	-.0465	-.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	-.0108	-.0311	-.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	-.0051	-.0232	-.0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	-.0021	-.0185	-.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0012	-.0119	-.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	-.0080	-.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	-.0058	-.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	-.0041	-.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	-.0022	-.0146
10.0	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	-.0012	-.0122
12.0	.0000	-.0001	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	-.0005	-.0104
14.0	.0000	.0000	.0000	.0000	+.0001	+.0008	+.0019	+.0023	-.0001	-.0090
16.0	.0000	.0000	-.0001	-.0002	-.0001	+.0004	+.0013	+.0019	+.0001	-.0079

Table VIII

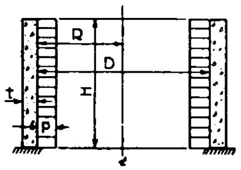


Moments in cylindrical wall
Trapezoidal load
Hinged base, free top

Mom. = coef. X $(wH^3 + pH^2)$ ft.lb. per ft.
Positive sign indicates tension in the outside

$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point									
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+.0329	+.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0171	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0155	0
2.0	+.0009	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0145	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0111	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0092	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0078	0
6.0	.0000	.0000	+.0002	+.0008	+.0019	+.0039	+.0062	+.0078	+.0068	0
8.0	.0000	.0000	-.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0054	0
10.0	.0000	.0000	-.0002	-.0001	+.0002	+.0011	+.0025	+.0043	+.0045	0
12.0	.0000	.0000	-.0001	-.0002	.0000	+.0005	+.0017	+.0032	+.0039	0
14.0	.0000	.0000	-.0001	-.0001	-.0001	.0000	+.0012	+.0026	+.0033	0
16.0	.0000	.0000	.0000	-.0001	-.0002	-.0004	+.0008	+.0022	+.0029	0

Table IX

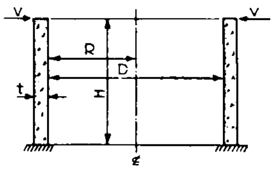


Moments in cylindrical wall
Rectangular load
Fixed base, free top

Mom. = coef. X pH^2 ft.lb. per ft.
Positive sign indicates tension in the outside

$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point									
0.4	-.0023	-.0093	-.0227	-.0439	-.0710	-.1018	-.1455	-.2000	-.2593	-.3310
0.8	.0000	-.0006	-.0025	-.0083	-.0185	-.0362	-.0594	-.0917	-.1325	-.1835
1.2	+.0008	+.0026	+.0037	+.0029	-.0009	-.0089	-.0227	-.0468	-.0815	-.1178
1.6	+.0011	+.0036	+.0062	+.0077	+.0068	+.0011	-.0093	-.0267	-.0529	-.0876
2.0	+.0010	+.0036	+.0066	+.0088	+.0089	+.0059	-.0019	-.0167	-.0389	-.0719
3.0	+.0007	+.0026	+.0051	+.0074	+.0091	+.0083	+.0042	-.0053	-.0223	-.0483
4.0	+.0004	+.0015	+.0033	+.0052	+.0068	+.0075	+.0053	-.0013	-.0145	-.0365
5.0	+.0002	+.0008	+.0019	+.0035	+.0051	+.0061	+.0052	+.0007	-.0101	-.0293
6.0	+.0001	+.0004	+.0011	+.0022	+.0036	+.0049	+.0048	+.0017	-.0073	-.0242
8.0	.0000	+.0001	+.0003	+.0008	+.0018	+.0031	+.0038	+.0024	-.0040	-.0184
10.0	.0000	-.0001	.0000	+.0002	+.0009	+.0021	+.0030	+.0026	-.0022	-.0147
12.0	.0000	.0000	-.0001	.0000	+.0004	+.0014	+.0024	+.0022	-.0012	-.0123
14.0	.0000	.0000	.0000	.0000	+.0002	+.0010	+.0018	+.0021	-.0007	-.0105
16.0	.0000	.0000	.0000	-.0001	+.0001	+.0006	+.0012	+.0020	-.0005	-.0091

Table X



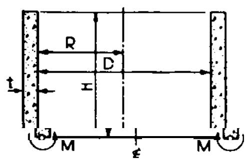
Moments in cylindrical wall
Shear per ft., V, applied at top
Fixed base, free top

Mom. = coef. X VH ft.lb. per ft.
Positive sign indicates tension in outside

$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.711	0.8H	0.9H	1.0H
	Coefficients at point*									
0.4	+0.093	+0.172	+0.240	to.300	+0.354	+0.402	+0.448	+0.492	+0.535	+0.578
0.8	+0.085	+0.145	+0.185	+0.208	co.220	+0.224	+0.223	+0.219	+0.214	+0.208
1.2	+0.082	+0.132	+0.157	+0.164	+0.159	+0.145	+0.127	+0.106	+0.084	+0.062
1.6	+0.079	+0.122	+0.139	+0.138	to.125	+0.105	+0.081	+0.056	+0.030	+0.004
2.0	+0.077	+0.115	+0.126	+0.119	+0.103	+0.080	+0.056	+0.031	+0.006	-0.019
3.0	+0.072	+0.100	+0.100	+0.086	+0.066	+0.044	+0.025	+0.006	-0.010	-0.024
4.0	+0.068	+0.088	+0.081	+0.063	co.043	+0.025	+0.010	-0.001	-0.010	-0.019
5.0	+0.064	+0.078	+0.067	to.047	+0.028	+0.013	+0.003	-0.003	-0.007	-0.011
6.0	+0.062	+0.070	+0.056	+0.036	+0.018	+0.006	0.000	-0.003	-0.005	-0.006
8.0	+0.057	+0.058	+0.041	co.021	+0.007	0.000	-0.002	-0.003	-0.002	-0.001
10.0	+0.053	+0.049	+0.029	to.012	+0.002	-0.002	-0.002	-0.002	-0.001	0.000
12.0	+0.049	+0.042	+0.022	to.007	0.000	-0.002	-0.002	-0.001	0.000	0.000
14.0	+0.046	+0.036	+0.017	to.004	-0.001	-0.002	-0.001	-0.001	0.000	0.000
16.0	+0.044	+0.031	+0.012	+0.001	-0.002	-0.002	-0.001	0.000	0.000	0.000

*When this table is used for shear applied at the base, while the top is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Table XI



Moments in cylindrical wall
Moment per ft., M, applied at base
Hinged base, free top

Mom. = coef. X M ft.lb. per ft.
Positive sign indicates tension in outside

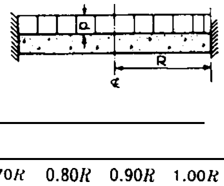
$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.911	1.0H
	Coefficients at point*									
0.4	+0.013	+0.051	+0.109	+0.196	+0.296	+0.414	+0.547	+0.692	+0.843	+1.000
0.8	+0.009	+0.040	+0.090	+0.164	+0.253	+0.375	+0.503	+0.659	+0.824	+1.000
1.2	+0.006	+0.027	+0.063	+0.125	+0.206	+0.316	+0.454	+0.616	+0.802	+1.000
1.6	+0.003	+0.011	+0.035	+0.078	+0.152	+0.253	+0.393	+0.570	+0.775	+1.000
2.0	-0.002	-0.002	+0.012	+0.034	+0.096	+0.193	+0.340	+0.519	+0.748	+1.000
3.0	-0.007	-0.022	-0.030	-0.029	+0.010	+0.087	+0.227	+0.426	+0.692	+1.000
4.0	-0.008	-0.026	-0.044	-0.051	-0.034	+0.023	+0.150	+0.354	+0.645	+1.000
5.0	-0.007	-0.024	-0.045	-0.061	-0.057	-0.015	+0.095	+0.296	+0.606	+1.000
6.0	-0.005	-0.018	-0.040	-0.058	-0.065	-0.037	+0.057	+0.252	+0.572	+1.000
8.0	-0.001	-0.009	-0.022	-0.044	-0.068	-0.062	+0.002	+0.178	+0.515	+1.000
0.0	0.000	-0.002	-0.009	-0.028	-0.053	-0.067	-0.031	+0.123	+0.467	+1.000
2.0	0.000	0.000	-0.003	-0.016	-0.040	-0.064	-0.049	+0.081	+0.424	+1.000
14.0	0.000	0.000	0.000	-0.008	-0.029	-0.059	-0.060	+0.048	+0.387	+1.000
16.0	0.000	0.000	+0.002	-0.003	-0.021	-0.051	-0.066	+0.025	+0.354	+1.000

*When this table is used for moment applied at the top, while the top is hinged. 0.0H is the bottom of the wall and 1.0H is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Table XII

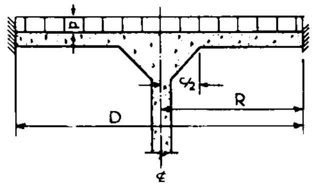
Moments in circular slab without center support
Uniform load
Fixed edge

Mom. = coef. X pR^2 ft.lb. per ft.
Positive sign indicates compression in surface loaded



).00R	0.10R	0.20R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
Coefficients at point										
Radial moments, M_r										
+.075	+.073	+.067	+.057	+.043	+.025	+.003	-.023	-.053	-.087	-.125
Tangential moments, M_t										
+.075	+.074	+.071	+.066	+.059	+.050	+.039	+.026	+.011	.006	-.025

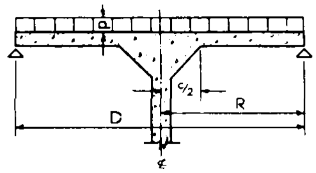
Table XIII



Moments in circular slab with center support
Uniform load
Fixed edge
Mom. = coef. X pR^2 ft.lb. per ft.
Positive sign indicates compression in surface loaded

c/D	Coefficients at point												
	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	-0.2100	-0.0729	-0.0275	-0.0026	+0.0133	+0.0238	+0.0342	+0.0347	+0.0277	+0.0142	-0.0049	-0.0294	-0.0589
0.10		-0.1433	-0.0624	-0.0239	-0.0011	+0.0136	+0.0290	+0.0326	+0.0276	+0.0158	-0.0021	-0.0255	-0.0541
0.15			-0.1089	-0.0521	-0.0200	+0.0002	+0.0220	+0.0293	+0.0269	+0.0169	+0.0006	-0.0216	-0.0490
0.20				-0.0862	-0.0429	-0.0161	to.0133	+0.0249	+0.0254	+0.0176	+0.0029	-0.0178	-0.0441
0.25					4.0698	-0.0351	+0.0029	+0.0194	+0.0231	10.0177	10.0049	-0.0143	-0.0393
	Tangential moments, M_t												
0.05	-0.0417	-0.0700	-0.0541	-0.0381	-0.0251	-0.0145	+0.0002	+0.0085	+0.0118	to.0109	+0.0065	-0.0003	-0.0118
0.10		-0.0287	-0.0421	-0.0354	-0.0258	-0.0168	-0.0027	+0.0059	+0.0099	+0.0098	to.0061	-0.0003	-0.0108
0.15			-0.0218	-0.0284	-0.0243	-0.0177	-0.0051	+0.0031	+0.0080	+0.0086	to.0057	-0.0006	-0.0098
0.20				-0.0172	-0.0203	-0.0171	-0.0070	+0.0013	+0.0063	to.0075	+0.0052	-0.0003	-0.0088
0.25					-0.0140	-0.0150	-0.0083	-0.0005	+0.0046	+0.0064	+0.0048	0.0000	-0.0078

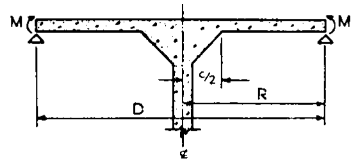
fable XIV



Moments in circular slab with center support
Uniform load
Hinged edge
Mom. = coef. X pR^2 ft.lb. per Ft.
Positive sign indicates compression in surface loaded

c/D	Coefficients at point												
	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	-0.3658	-0.1388	-0.0640	-0.0221	+0.0058	+0.0255	+0.0501	+0.0614	to.0629	+0.0566	+0.0437	+0.0247	0
0.10		-0.2487	-0.1 180	-0.0557	-0.0176	+0.0081	+0.0391	+0.0539	+0.0578	+0.0532	+0.0416	+0.0237	0
0.15			-0.1869	-0.0977	-0.0467	-0.0135	+0.0258	+0.0451	+0.0518	+0.0494	+0.0393	+0.0226	0
0.20				-0.1465	-0.0800	-0.0381	+0.0109	+0.0352	+0.0452	+0.0451	+0.0368	10.0215	0
0.25					-0.1172	-0.0645	-0.0055	+0.0245	+0.0381	+0.0404	+0.0340	+0.0200	0
	Tangential moments, M_t												
0.05	-0.0731	-0.1277	-0.1040	-0.0786	-0.0569	-0.0391	-0.0121	+0.0061	+0.0175	+0.0234	+0.0251	+0.0228	+0.0168
0.10		-0.0498	-0.0768	-0.0684	-0.0539	-0.0394	-0.0153	+0.0020	10.0134	+0.0197	+0.0218	+0.0199	+0.0145
0.15			-0.0374	-0.0516	-0.0470	-0.0375	-0.0175	-0.0014	+0.0097	+0.0163	+0.0186	+0.0172	+0.0123
0.20				-0.0293	-0.0367	-0.0333	-0.0184	-0.0042	+0.0065	+0.0132	+0.0158	+0.0148	+0.0103
0.25					-0.0234	-0.0263	-0.0184	-0.0062	+0.0038	+0.0103	+0.0132	+0.0122	+0.0085

Table XV



Moments in circular slab with center support
Moment per ft., **M,** applied at edge
Hinged edge
Mom. = coef. X M ft.lb. per Ft.
Positive sign indicates compression in top surface

c/D	Coefficients at point												
	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.5011	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	-2.650	-1.121	-0.622	-0.333	-0.129	+0.029	+0.268	to.450	+0.596	+0.718	to.824	+0.917	+1.000
0.10		-1.950	-1.026	-0.584	-0.305	-0.103	+0.187	+0.394	+0.558	+0.692	+0.808	to.909	11.000
0.15			-1.594	-0.930	-0.545	-0.280	+0.078	+0.323	+0.510	+0.663	+0.790	to.900	+1.000
0.20				-1.366	-0.842	-0.499	-0.057	+0.236	+0.451	+0.624	+0.768	+0.891	+1.000
0.25					-1.204	-0.765	-0.216	+0.130	+0.392	10.577	+0.740	10.880	+1.000
	Tangential moments, M_t												
0.05	-0.530	-0.980	-0.847	-0.688	-0.544	-0.418	-0.211	-0.042	+0.095	to.212	+0.314	+0.405	+0.486
0.10		-0.388	-0.641	-0.608	-0.518	-0.419	-0.233	-0.072	10.066	+0.185	+0.290	+0.384	+0.469
0.15			-0.319	-0.472	-0.463	-0.404	-0.251	-0.100	+0.035	10.157	+0.263	+0.363	to.451
0.20				-0.272	-0.372	-0.368	-0.261	-0.123	+0.007	+0.129	+0.240	+0.340	to.433
0.25					-0.239	-0.305	-0.259	-0.145	-0.020	to.099	to.714	+0.320	to.414

Table XVI

Shear at base of cylindrical wall

$$V = \text{coef.} \times \begin{cases} wH^2 \text{ lb. (triangular)} \\ pH \text{ lb. (rectangular)} \\ M/H \text{ lb. (mom. at base)} \end{cases}$$

Positive sign indicates shear acting inward

$\frac{H^2}{Dt}$	Triangular load, fixed base	Rectangular load, fixed base	Triangular or rectangular load, hinged base	Moment at edge
0.4	+0.436	+0.755	+0.245	-1.58
0.8	+0.374	+0.552	+0.234	-1.75
1.2	+0.339	+0.460	+0.220	-2.00
1.6	+0.317	+0.407	+0.204	-2.28
2.0	+0.299	+0.370	+0.189	-2.57
3.0	+0.262	+0.310	+0.158	-3.18
4.0	+0.236	+0.271	+0.137	-3.68
5.0	+0.213	+0.243	+0.121	-4.10
6.0	+0.197	+0.222	+0.110	-4.49
8.0	+0.174	+0.193	+0.096	-5.18
10.0	+0.158	+0.172	+0.087	-5.81
12.0	+0.145	+0.158	+0.079	-6.38
14.0	+0.135	+0.147	+0.073	-6.88
16.0	+0.127	+0.137	+0.068	-7.36

Table XVIII

Stiffness of cylindrical wall
Near edge hinged, far edge free
$k = \text{coef.} \times Et^3/H$

$\frac{H^2}{Dt}$	Coefficient	$\frac{H^2}{Dt}$	Coefficient
		5	0.713
		6	0.783
		8	0.903
[illegible]	[illegible]	10	1.010
2.0	0.445	12	1.108
3.0	0.548	14	1.198
4.0	0.635	16	1.281

Table XVII

Load on center support for circular slab

$$\text{Load} = \text{coef.} \times \begin{cases} R^2 \text{ (hinged and fixed)} \\ M \text{ (moment at edge)} \end{cases}$$

c/D	0.05	0.10	0.15	0.20	0.25
Hinged	1.320	1.387	1.463	1.542	1.625
Fixed	0.839	0.919	1.007	1.101	1.200
M at edge	8.16	8.66	9.29	9.99	10.81

Table XIX

Stiffness of circular plates
With center support
$k = \text{coef.} \times Et^3/R$

c/D	0.05	0.10	0.15	0.20	0.25
Coef.	0.290	0.309	0.332	0.358	0.387

Without center support
Coef. = 0.104

Table XX. Supplementary Coefficients for Values of H^2/Dt Greater than 16 (Extension of Tables I to XI, XVI and XVIII)*

$\frac{H^2}{Dt}$	TABLE I — Coefficients at point					TABLE II — Coefficients at point				
	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H
20	+0.716	+0.654	+0.520	+0.325	+0.115	+0.812	+0.817	+0.756	+0.603	+0.344
24	+0.746	+0.702	+0.577	+0.372	+0.137	+0.816	+0.839	+0.793	+0.647	+0.377
32	+0.782	+0.768	+0.663	+0.459	+0.182	+0.814	+0.861	+0.847	+0.721	+0.436
40	+0.800	+0.805	+0.731	+0.530	+0.217	+0.802	+0.866	+0.880	+0.778	0.483
48	+0.791	+0.828	+0.785	+0.593	+0.254	+0.791	+0.864	+0.900	+0.820	+0.527
56	+0.763	+0.838	+0.824	+0.636	+0.285	+0.781	+0.859	+0.911	+0.852	+0.563

$\frac{H^2}{Dt}$	TABLE III — Coefficients at point					TABLE IV — Coefficients at point				
	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H
20	+0.949	+0.825	+0.629	+0.379	+0.128	+1.062	+1.017	+0.906	+0.703	+0.394
24	+0.986	+0.879	+0.694	+0.430	+0.149	+1.066	+1.039	+0.943	+0.747	+0.427
32	+1.026	+0.953	+0.788	+0.519	+0.189	+1.064	+1.061	+0.997	+0.821	+0.486
40	+1.040	+0.996	+0.859	+0.591	+0.226	+1.052	+1.066	+1.030	+0.878	+0.533
48	+1.043	+1.022	+0.911	+0.652	+0.262	+1.041	+1.064	+1.050	+0.920	+0.577
56	+1.040	+1.035	+0.949	+0.705	+0.294	+1.021	+1.059	+1.061	+0.952	+0.613

$\frac{H^2}{Dt}$	TABLE V — Coefficients at point					TABLE VI — Coefficients at point				
	.00H	.05H	.10H	.15H	.20H	.75H	.80H	.85H	.90H	.95H
20	-18.44	- 9.98	- 4.90	- 1.59	+ 0.22	+15.30	+ 25.9	+ 36.9	+ 43.3	+ 35.3
24	-18.04	-10.34	- 4.54	- 1.00	+ 0.68	+13.20	+ 25.9	+ 40.7	+ 51.8	+ 45.3
32	-20.84	-10.72	- 3.70	- 0.04	+ 1.26	+ 8.10	+ 23.2	+ 45.9	+ 65.4	+ 63.8
40	-23.34	-10.86	- 2.86	+ 0.72	+ 1.56	+ 3.28	+ 19.2	+ 46.5	+ 77.9	+ 83.5
46	-25.52	-10.82	2.06	+ 1.26	+ 1.66	0.70	+ 14.1	+ 45.1	+ 87.2	+103.0
56	-27.54	-10.68	1.36	+ 1.60	+ 1.62	- 3.40	+ 9.2	+ 42.2	+ 94.0	+121.0

$\frac{H^2}{Dt}$	TABLE VII — Coefficients at point					TABLE VIII — Coefficients at point				
	.80H	.85H	.90H	.95H	1.00H	.75H	.80H	.85H	.90H	.95H
20	+.0015	+.0014	+.0005	-.0018	-.0063	+.0008	+.0014	+.0020	+.0024	+.0020
24	+.0012	+.0012	+.0007	-.0013	-.0053	+.0005	+.0010	+.0015	+.0020	+.0017
32	+.0007	+.0009	+.0007	-.0008	-.0040	.0000	+.0005	+.0009	+.0014	+.0013
40	+.0002	+.0005	+.0006	-.0005	-.0032	.0000	+.0003	+.0006	+.0011	+.0011
46	.0000	+.0001	+.0006	-.0003	-.0026	.0000	+.0001	+.0004	+.0008	+.0010
56	.0000	.0000	+.0004	-.0001	-.0023	.0000	.0000	+.0003	+.0007	+.0008

$\frac{H^2}{Dt}$	TABLE IX — Coefficients at point					TABLE X — Coefficients at point				
	.80H	.85H	.90H	.95H	1.00H	.05H	.10H	.15H	.20H	.25H
20	+.0015	+.0013	+.0002	-.0024	-.0073	+0.032	+0.039	+0.033	+0.023	+0.014
24	+.0012	+.0012	+.0004	-.0018	-.0061	+0.031	+0.035	+0.028	+0.018	+0.009
32	+.0008	+.0009	+.0006	-.0010	-.0046	+0.028	+0.029	+0.020	+0.011	+0.004
40	+.0005	+.0007	+.0007	-.0005	-.0037	+0.026	+0.025	+0.015	+0.006	+0.001
48	+.0004	+.0006	+.0006	-.0003	-.0031	+0.024	+0.021	+0.011	+0.003	0.000
5 8	+.0002	+.0004	+.0005	-.0001	-.0026	+0.023	+0.018	+0.008	+0.002	0.000

$\frac{H^2}{Dt}$	TABLE XI — Coefficients at point					TABLE XVI				TABLE XVIII
	.80H	.85H	.90H	.95H	1.00H	Tri. Fixed	Rect. Fixed	T. or R. Hinged	Mom. at Edge	Stiffness
20	-0.015	+0.095	+0.296	+0.606	+1.000	+0.114	+0.122	+0.062	- 8.20	1.430
24	-0.037	+0.057	+0.250	+0.572	+1.000	+0.102	+0.111	+0.055	- 8.94	1.566
32	-0.062	+0.002	+0.178	+0.515	+1.000	+0.089	+0.096	+0.048	-10.36	1.810
40	-0.067	-0.031	+0.123	+0.467	+1.000	+0.080	+0.086	+0.043	-11.62	2.025
48	4.064	-0.049	+0.081	+0.424	+1.000	+0.072	+0.079	+0.039	-12.76	2.220
5 8	-0.059	-0.060	+0.048	+0.387	+1.000	+0.067	+0.074	+0.036	-13.76	2.400

*For points not shown in the supplementary tables, ring tension and moment may be determi[illegible]ed approximately by sketching curves simila[illegible] those in the text.

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