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Comparison between analog and digital simulation techniques for aquifer evaluation

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ABSTRACT: Several comparisons are made between electric analog and digital computer techniques in the evaluation of aquifer systems. The analog method described involves electronic analyzers coupled to an array of electrical resistors and capacitors simulating a scaled-down version of the aquifer. The digital solutions described are obtained by implicit numerical integration.

Comparisons are first made by noting the similarities between the two computational methods. The most important similarities include basic data requirements, method of discretizing the space variables, assigning hydrogeologic properties to the discrete portions of the model, verification of the model, and analysis of the basic problem.

On the basis of the study it was found that the main differences between the analog and digital computational techniques fall basically into the four categories as follows: 1) The digital method does not require as much time for model construction and data readout and is more convenient than the equipment manipulation phases needed in the analog technique; 2) The digital computer offers more versatility than the analog for solving nonlinear boundary problems; 3) The analog technique offers the advantage of an in-house simulator for a large class of groundwater problems and when it is desired to solve problems requiring an insight into the behavior of the physical system; and 4) The analog technique offers advantages when solving large problems.

Factors related to accessibility of equipment and costs dictate to a large extent the simulation technique choice.

RÉSUMÉ : Diverses comparaisons sont faites entre les techniques des calculateurs analogiques et digitaux pour l'évaluation de systèmes hydrauliques. La méthode par analogie décrite comprend des analyseurs électroniques couplés à un ensemble de résistances et de capacités électriques simulant une version de la nappe. Les solutions par calculateur digital décrites sont obtenues par l'intégration implicite numérique.

Des comparaisons sont d'abord faites en notant les similitudes entre les deux méthodes. Les plus importantes similitudes comprennent la nécessité de la connaissance de données de base, la méthode pour rendre les variables spaciales discrètes, l'assignation de propriétés hydrogéologiques aux portions discrètes du modèle, la vérification du modèle et l'analyse du problème de base.

Cette étude montre d'ailleurs que les plus grandes différences entre les techniques analogiques et digitales tombent dans l'une des quatre catégories suivantes :

1) La méthode digitale ne demande pas autant de temps pour la construction du modèle et elle convient mieux que les multiples phases de manipulation nécessitées par la technique analogique;
2) La méthode digitale offre plus de possibilités que l'analogie pour la solution des problèmes à limites non linéaires;
3) La technique par simulation offre l'avantage d'une simulation pour une large classe de problèmes d'eaux souterraines, et, quand cela est désiré, permet de résoudre des problèmes exigeant une vue sur le comportement du système physique;
4) La méthode analogique est avantageuse pour la solution de problèmes étendus.

Des facteurs se rapportant à la possibilité de disposer de l'équipement ainsi qu'aux frais dictent dans une large mesure le choix de la technique de simulation.

INTRODUCTION

The groundwater hydrologist is concerned chiefly with the quantitative description of aquifers and their response to development. Proper planning in the development of groundwater basins require aquifer evaluations that include testing of possible schemes and appraising the relative merits of various alternatives. Primarily, the questions pertaining to the use of groundwater resources requires that pumping be related to water-level change with reference to both time and space. The factors to be considered in aquifer evaluation are the cause and effect—pumpage and changes in water levels.

The groundwater hydrologist has four basic types of analytical tools that can be used for aquifer evaluation as follows: 1) the simple image well theory applied to a highly idealized aquifer system; 2) electric analog simulators, involving electronic analyzers coupled to an array of resistors and capacitors simulating a scaled-down version of the aquifer; 3) the general-purpose digital computer; and 4) the analog-digital combination forming a hybrid computer. Under suitable circumstances, each type of analytical tool has its place in studying the response of groundwater systems to pumpage.

This paper concerns comparisons made between the electric analog simulator and the digital computer as tools in aquifer evaluation. The main part of the comparison is made under the special condition that the problem can be solved by either device. This seems to be a realistic approach for reducing the comparison problem that involves many variables. Comparisons are made on that basis for several types of identical aquifer systems. Several other types of constant parameter aquifers and attendant nonlinear boundary conditions such as induced infiltration, evapotranspiration phenomena, and stepwise-time varying pumpage from wells were studied and compared.

Some special difficulties in modeling boundary conditions such as saltwater intrusion, nonlinear transmissivity, and large problems are briefly discussed.

The paper has been divided into four major parts. For the purpose of illustration, the first and second parts describe a common type of electric analog simulator and a digital programming technique that can be used in aquifer evaluation. The third section includes an outline of the particular types of aquifers and boundary conditions compared. The last section of the paper discusses the comparisons and some special problem areas.

Hopefully, the results of this study may clarify some of the relative merits of the electric analog and digital computer simulation techniques in aquifer evaluation.

ANALOG SIMULATION TECHNIQUE

Most electric analog simulators used in evaluating aquifers under artesian, nonsteady-state, two-dimensional flow conditions are basically designed as illustrated in figure 1. The design method outlined includes procedures for simulating the aquifer and simultaneously pumping any number of wells at differing constant rates for a given continuous time period.

The simple analog simulator shown in figure 1 is of the discrete space, continuous time type and is made up of an electronic analyzer coupled to an analog model. The analog model consists of an array of electrical resistors and capacitors simulating a scaled-down version of the aquifer. The resistor values are made inversely proportional to the aquifer transmissivity and the capacitor values are designed to be directly proportional to the aquifer storage properties. The behavior of the electrical network is described by equations that have the same form as the finite-difference equations for nonsteady-state, two-dimensional flow of groundwater (Skibitzke, 1960). Electrical units (volts, coulombs, amperes, and seconds) and corresponding hydraulic units (feet, gallons, gallons per day, and days) are connected by scale factors.

The pulse and waveform generator components of the electronic analyzer forces electrical current to flow in the analog model in the proper time sequence. The oscilloscope measures the resulting time-variant potential levels of the analog model. The oscilloscope traces (time-voltage graphs) are analogous to time-drawdown graphs.

Many complex boundary conditions can be incorporated as extensions of the basic simulator illustrated. The presentation of specific theory and design methods for the complex boundary conditions are beyond the scope of this paper. Detailed discussions of design methods and analog computational techniques applied to groundwater reser-

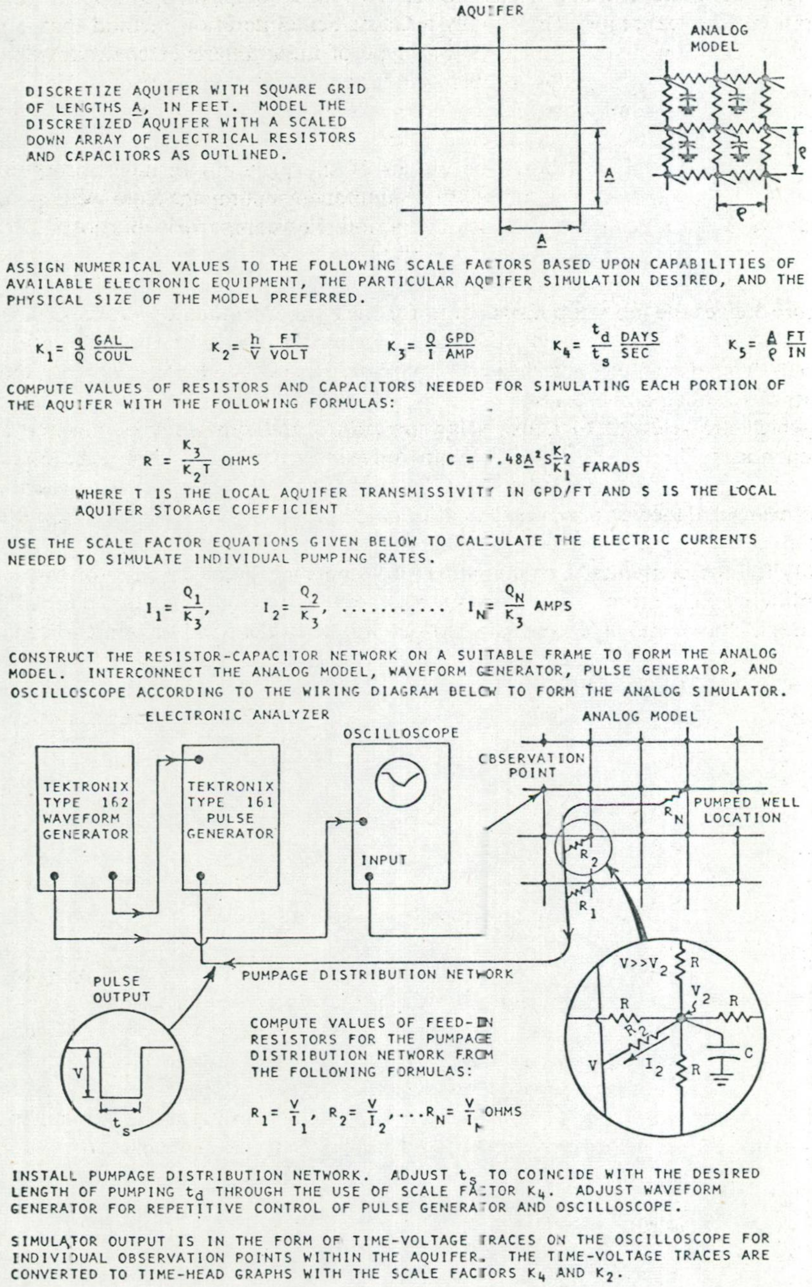


FIGURE 1. *Example design of an electric analog simulator*

voirs can be obtained from Karplus (1958), Walton and Prickett (1963), Skibitzke (1963), Patten (1965), and Prickett (1967).

DIGITAL SIMULATION TECHNIQUE

The digital computer simulations of this report were accomplished by implicit numerical integration. The technique is essentially a Gauss-Seidel iteration method that is used to solve a set of discrete space, discrete time type of finite-difference equations applied to the particular groundwater flow problem under study (Tyson and Weber, 1964).

An example of a simple computer program is given in figure 2. This program is written in FORTRAN IV and can be used in evaluating aquifers under artesian, two-dimensional, nonsteady-state flow conditions. The design of the program includes procedures for simulating the aquifer and accommodating simultaneous pumpage from wells at different constant rates for a given continuous time period. For comparison purposes, the digital computer program in figure 2 is set up to solve the same type of problem illustrated in the electric analog simulator design of figure 1.

Figure 3 gives the job setup instructions for the aquifer simulation program of figure 2. Briefly, the aquifer is subdivided into discrete portions associated with nodes and branches. The attendant node cards contain data pertinent to a water balance that must exist in the aquifer domain assigned to the individual nodes. The branch cards contain data which are relevant to Darcy's law describing the flow of groundwater between adjacent nodes. The parameter card is set up to give information concerning the discrete time steps to be simulated, the largest node and branch numbers used, the maximum allowable sum of water balance errors over the entire model, and the relaxation coefficient used to speed up convergence. The total data deck is placed behind the program deck and the job is ready for the computer. The computer program then prepares and solves the set of finite-difference equations governing groundwater flow interaction between the nodes and branches of the model. The computer output will be in the form of printed heads for all nodes at the end of every time increment.

Detailed discussions on several types of numerical techniques, special nonlinear boundary conditions, and possible computer output in the form of graphs and maps are available in Chun *et al.* (1964, 1967), Remson *et al.* (1965), Bittinger *et al.* (1967), and Longenbaugh (1967).

SELECTED SIMULATIONS

Several comparisons were made between electric analog and digital computer techniques in evaluating aquifer systems. The following types of constant-parameter aquifers were selected for study: nonleaky, leaky, and multilayered aquifers in one, two, and three dimensions with both uniform and non-uniform grid spacings. In addition to studying the types of aquifers outlined above, the following special boundary condition techniques were compared: irregular impermeable boundaries, pumpage from wells, irregular shaped rivers involving nonlinear properties associated with induced infiltration, the nonlinear storage coefficients attendant with the conditions of converting from artesian to water-table conditions under heavy pumping, and nonlinear types of recharge in the form of recoverable evapotranspiration losses, precipitation, underflow, and excess irrigation.

Two groups of comparative models were analyzed. The first group of models were of idealized groundwater basins wherein analog, digital and theoretical solutions could be compared. The analog and theoretical models were available from a previous study by Walton and Prickett (1963). The second group of comparative models consisted of theoretical aquifers incorporating individually simulated special boundary conditions.

In addition a 1500 node digital model of an actual field problem that had been previously solved in an analog solution by Schicht (1965) was also constructed and analyzed. This particular field problem involved an aquifer evaluation of the single layered sand and gravel aquifer in the area of East St. Louis, Illinois.

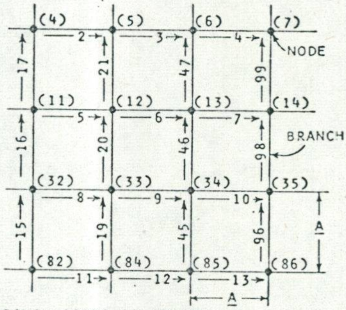
The analog simulations were carried out with existing electronic equipment at the Illinois State Water Survey. The digital simulations were accomplished with Water Survey

```
C      AQUIFER SIMULATION PROGRAM V3
C      ILLINOIS STATE WATER SURVEY
C
C      DEFINITION OF VARIABLES
C
C      MAXI-------HIGHEST NODE NUMBER
C      MAXJ-------HIGHEST BRANCH NUMBER
C      DELTA------TIME INCREMENT (DAYS)
C      NSTEPS-----NO. OF TIME INCREMENTS
C      RC---------RELAXATION COEFFICIENT
C      ERROR------TOLERANCE LEVEL OF
C                 TOTAL WATER BALANCE
C                 ERRORS (GAL/DAY)
C      ADJUST-----ADJUSTMENT COEFFICIENT
C                 (FT/(GAL/DAY))
C      TRANS(J)---AQUIFER TRANSMISSIVITY
C                 (GAL/DAY*FT)
C      AQ(I)------PUMPING RATES (GAL/DAY)
C      Q(J)-------BRANCH FLOW RATE
C                 (GAL/DAY)
C      SFLOW------STORAGE FLOW RATE
C                 (GAL/DAY)
C      AS(I)------STORAGE FACTOR (GAL)
C      HO(I)------INITIAL HEADS (FT)
C      H(I)-------HEADS (FT)
C      RES(I)-----WATER BALANCE ERROR
C                 (GAL/DAY)
C      NODE(I,K)--LIST OF BRANCHES
C                 CONNECTED TO NODES
C      IBRAN(J,K)-NODE NUMBERS OF BRANCH
C                 ENDPOINTS
C      IN THE ABOVE DEFINITIONS
C      I----------SIGNIFIES NODE NUMBER
C      J----------SIGNIFIES BRANCH NUMBER
C
C
C
C
C
       DIMENSION TRANS(3000).AQ(1500),
      1 Q(3000),AS(1500),HO(1500)
      2 ,H(1500),RES(1500),NODE(1500,4),
      3 IBRAN(3000,2),ADJUST(1500)
C
C      READ PARAMETER CARD
C
 10    READ(5,20)DELTA,NSTEP,MAXI,MAXJ,
      1 ERROR,RC
 20    FORMAT(F5.0,3I5,2F10.0)
C
C      CLEAR STORAGE ARRAYS
C
       TIME=0.0
       DO 30 I=1,MAXI
       AQ(I)=0.0
       H(I)=0.0
       AS(I)=0.0
       DO 30 K=1,4
 30    NODE(I,K)=0
       DO 40 J=1,MAXJ
       IBRAN(J,1)=0
       IBRAN(J,2)=0
 40    TRANS(J)=0.0
C
C      READ NODE CARDS
C
 50    READ(5,60)I,(NODE(I,K),K=1,4),H(I)
      1,AQ(I),AS(I)
 60    FORMAT(5I5,3F10.0)
       IF(I-MAXI)50,70,70
C
C      READ BRANCH CARDS
C
 70    READ(5,80)J,TRANS(J),IBRAN(J,1),
      1 IBRAN(J,2)
 80    FORMAT(I5,F10.0,2I5)
       IF(J-MAXJ)70,90,90
C      CALCULATE ADJUSTMENT COEFFICIENTS
C
 90    DO 130 I=1,MAXI
C      SUM TRANSMISSIVITIES OF BRANCHES
       SUMT=0.0
       DO 120 K=1,4
       L=NODE(I,K)
       IF(L)100,120,110
 100   L=-L
 110   SUMT=SUMT+TRANS(L)
 120   CONTINUE
 130   ADJUST(I)=RC/(SUMT+AS(I)/DELTA)
C
C      START OF SIMULATION
C
       DO 250 M=1,NSTEP
       TIME=TIME+DELTA
C
C      INITIALIZE HO(I)
C
       DO 140 I=1,MAXI
 140   HO(I)=H(I)
C
C      CALCULATE BRANCH FLOW RATES
C
 150   DO 170 J=1,MAXJ
       IF(TRANS(J))170,170,160
 160   K=IBRAN(J,1)
       KK=IBRAN(J,2)
       Q(J)=TRANS(J)*(H(K)-H(KK))
 170   CONTINUE
       SERROR=0.0
       DO 210 I=1,MAXI
C
C      CALCULATE STORAGE FLOW RATES
C
       SFLOW =AS(I)*(H(I)-HO(I))/DELTA
C
C      SUM BRANCH FLOW RATES
C
       BFLOW=0.0
       DO 200 K=1,4
       J=NODE(I,K)
       IF(J)180,200,190
C      NEG NUMBER REPRESENTS OUTWARD FLOW
 180   BFLOW=BFLOW-Q(-J)
       GO TO 200
 190   BFLOW=BFLOW+Q(J)
 200   CONTINUE
C
C      CALCULATE WATER BALANCE ERRORS
C
       RES(I)=BFLOW-SFLOW-AQ(I)
 210   SERROR=SERROR+ABS(RES(I))
C
C      ADJUST HEADS
C
       DO 220 I=1,MAXI
 220   H(I)=H(I)+ADJUST(I)*RES(I)
C
C      CHECK SUM OF ERRORS AGAINST
C      CHOSEN TOLERANCE
C
       IF(SERROR-ERROR)230,150,150
C
C      PRINT RESULTS
C
 230   WRITE(6,240)TIME
 240   FORMAT(7H1TIME =F10.2)
 250   WRITE(6,260)(I,H(I),I=1,MAXI)
 260   FORMAT(10(I5,F8.3))
C
C      RESTART PROGRAM FOR
C      NEXT SIMULATION
C
       GO TO 10
       END
```

FIGURE 2. Example aquifer simulation program

data processing equipment and an IBM 7094 or an IBM System 360/50-75 computer. IBM 7094 and 360 computer time was purchased through the facilities of the Univeristy of Illinois in Urbana for \$ 90 and \$ 300 per hour respectively.

DISCRETIZE AQUIFER WITH SQUARE GRID OF LENGTHS A, IN FEET. ASSIGN NUMBERS TO NODES AND BRANCHES. ASSUME DIRECTIONS OF GROUNDWATER FLOW.



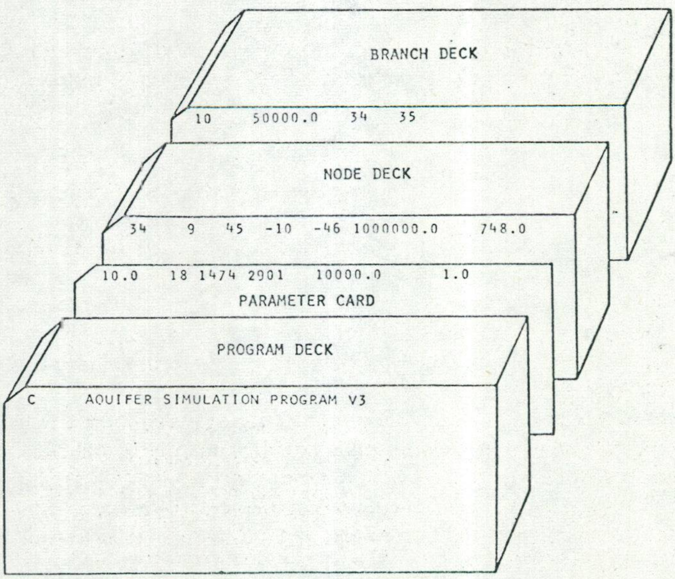
PREPARE PARAMETER CARD, NODE CARDS, AND BRANCH CARDS AS FOLLOWS:

PARAMETER CARD--
ENTER NUMERICAL VALUES FOR DELTA, NSTEP, MAX I, MAX J, ERROR, AND RC ACCORDING TO FORMAT STATEMENT 20.

NODE CARDS--
THE NODE DECK CONTAINS ONE CARD FOR EACH NODE. ENTER NODE NUMBER, ONE OR MORE BRANCH NUMBERS, INITIAL HEAD, NET WITHDRAWAL OR PUMPING RATE, AND THE LOCAL STORAGE FACTOR ON EACH CARD ACCORDING TO FORMAT STATEMENT 60. LET THE STORAGE FACTOR EQUAL $7.48A^2S$, WHERE S IS THE AQUIFER STORAGE COEFFICIENT. THE ENTERED BRANCH NUMBERS SHOULD BE PRECEDED BY A SIGN ACCORDING TO ASSUMED DIRECTIONS OF GROUNDWATER FLOW. FLOW INTO A NODE IS POSITIVE AND FLOW OUT OF A NODE IS NEGATIVE.

BRANCH CARDS--
THE BRANCH DECK CONTAINS ONE CARD FOR EACH BRANCH. ENTER THE BRANCH NUMBER, LOCAL AQUIFER TRANSMISSIVITY, AND TWO BRANCH ENDPOINT NODE NUMBERS ACCORDING TO FORMAT STATEMENT 80. THE PROGRAM IS WRITTEN SUCH THAT THE ASSUMED DIRECTION OF GROUNDWATER FLOW IS FROM THE FIRST INTO THE SECOND NODE NUMBER ENTERED ON THE BRANCH CARD.

PLACE PUNCHED PROGRAM, PARAMETER CARD, NODE AND BRANCH DECKS IN THE ORDER ILLUSTRATED BELOW. INCLUDE APPROPRIATE JOB CONTROL CARDS.



COMPUTER OUTPUT WILL BE IN THE FORM OF PRINTED HEADS FOR ALL NODES AT THE END OF EVERY TIME INCREMENT ACCORDING TO PROGRAM FORMAT STATEMENTS 240 AND 260.

FIGURE 3. *Setup instructions for example aquifer simulation program*

Availability of equipment was not studied, however it should be noted that this factor may be the reason for choosing between simulation techniques. An on-site investigation, mainly an economic one, should be made to determine availability.

COMPARISON OF SIMULATION TECHNIQUES

Comments regarding the comparisons of simulation techniques are grouped into the main categories of similarities, model design, model construction, nonlinear boundary simulation, data readout and display, factors related to costs, and special problem areas.

Both simulation techniques involved the same basic field data, the method of discretizing the space variables, and the same task of assigning hydrogeologic properties to the discrete portions of the model. Similarly, each simulation technique required familiarity with specialized vocabulary and equipment. In the case of the digital computer it was FORTRAN IV or some related program language that was applied to the particular computer used. The analog simulations required an association with electronic terminology and equipment. There were no differences in the model verification or analysis stages of accompanying either technique. The analog and digital techniques were both capable of exceeding the accuracy of available field data.

The digital technique does not require the manipulation of scale factors as in the analog technique, where mating of hydrogeologic parameters to the electronic equipment and model components is required.

In the category of model construction, the digital models were built in less time than the electric analog models. Essentially, the electric analog model construction requires placing and soldering of resistors and capacitors at every node plus possible connections associated with simulating special boundary conditions. On the otherhand, the digital models were constructed by the much less time-consuming process of specifying node and branch identifications and of keypunching data on cards. It was observed that, with the methods outlined, time involved in digital model construction was approximately one-half of that needed for electric analog model construction. It should be mentioned that special computer techniques, not discussed herein, can be applied to automate and thus further reduce the time for digital model construction.

On the basis of the comparisons made it was concluded that the digital computer can simulate nonlinear input conditions with greater ease and in a more convenient fashion than the electric analog techniques allow. An explanation of the problems involved in simulation of stepwise time-varying pumpage can be used as an illustration of this point.

A typical aquifer evaluation analysis might be accomplished with 6 incremental changes in pumpage from several hundred wells or several well fields each pumping at different rates (Patten, 1965). This can be accomplished with the analog simulator design given in figure1 by a time-consuming process of superposition. What is more generally done is to provide 6 pulse generators each driving a differently designed pumpage distribution network in sequence. On the otherhand, the procedures outlined in figure 4 show the changes that can be made to the basic digital program of figure 2 to include the type of time-varying pumpage in question. Briefly, pump cards are included in the data deck which define the desired node number locations of the wells and the time distribution of pumpage. Thepump cards are placed following the branch deck and the program is run.

In the digital approach above, the process of writing down the pumping scheme in a prescribed form and keypunching cards replaces the scale factor and equipment manipulations, resistor value calculations, and wiring the distribution networks needed in the analog technique.

Digital programming is similarly convenient compared to electric analog simulation techniques when nonlinear boundary condition modeling requires use of banks of diodes or field effect transistors as may by necessary, for example, in induced infiltration and evapotranspiration studies.

Data outputs in aquifer evaluation studies are usually arranged in the form of tabulations, piezometric surface contour maps, water balances, and graphs showing time variation of pumpage and water levels. In most cases, but not all, the analog simulation technique was much more time consuming in providing the needed output compared to the digital computer techniques. In the analog technique, data ware systematically extracted from selected model points as the analysis proceeded. Oscilloscope time-voltage traces were converted to time-water level change graphs through the use of scale factors. Then pertinent data were tabulated, plotted and contoured for the desired analysis.

INSERT THESE ADDITIONAL TERMS WITHIN THE DEFINITION OF VARIABLES SECTION

```
C      NPUMP------N NODE NUMBERS WHERE
C                 PUMPAGE TAKES PLACE
C      PUMPRT-----PUMPING RATES
C      N----------NUMBER OF PUMPS
```

ADD A FOURTH CONTINUATION CARD IN THE DIMENSION STATEMENT AS FOLLOWS

```
     4 ,NPUMP(100),PUMPRT(100,20)
```

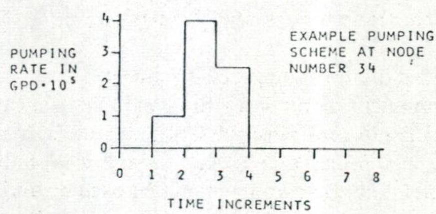
DELETE THE IF STATEMENT IMMEDIATELY BELOW STATEMENT 80 AND INSERT THE FOLLOWING INSTRUCTIONS

```
       IF(J-MAXJ)70,81,81
C
C      READ PUMP CARDS
C
81     N=1
82     READ(5,83,END=84)NPUMP(N),
     1 (PUMPRT(N,L),L=1,NSTEP)
83     FORMAT(I5,7F10.0/(5X7F10.0))
       N=N+1
       GO TO 82
84     N=N-1
C
```

DELETE ALL OF THE STATEMENTS BETWEEN THE COMMENT CARDS 'CALCULATE WATER BALANCE ERRORS' AND 'ADJUST HEADS' AND REPLACE THEM WITH THE FOLLOWING SET OF INSTRUCTIONS

```
C
210    RES(I)=BFLOW-SFLOW
       DO 215 L=1,N
       LL=NPUMP(L)
215    RES(LL)=RES(LL)-PUMPRT(L,M)
       DO 219 I=1,MAXI
219    SERROR=SERROR+ABS(RES(I))
C
```

PREPARE PUMP CARDS ACCORDING TO FORMAT STATEMENT 83 AS ILLUSTRATED



EXAMPLE PUMP CARD FOR NODE NUMBER 34

```
   34     0.0  100000.0  400000.0  250000.0       0.0       0.0
```

FIGURE 4. *Computer program modification for simulating stepwise time-varying pumpage*

The digital computer can be programmed to provide printout of data. For instance, the computer program of figure 2 contains the PRINT RESULTS section for automatic tabulation of time and water level data for all node points. Special computer programs have been written to automatically form the maps, water balances, and graphs and make them available as the direct computer output (Longenbaugh, 1967). This implied efficiency, however, had to be somewhat discounted in the studies herein because of sometimes lengthy job turnaround time involved in routing the job to the computer and back. Job turnaround time varied widely from 2 hours to over 24 hours and averaged about 8 hours.

A detailed cost analysis was not made but the following items seem to be pertinent. The electronic analyzer shown in figure 1 was purchased for approximately $ 1000. Analog model cost was additional and varied widely dependent upon the desired size of the model and the particular boundary problem under study. For instance, the East St. Louis analog model component cost was approximately $ 200, took about 6 man-days to construct, and required about 2 man-days to accomplish a typical cause and effect relationship involving the entire model. Under similar conditions the digital model card identification and keypunching data processes required about 3 man-days of time. In simulating 60 time steps, the 1500 node East St. Louis computer model typically consumed less than 6 minutes of 360 time per program run ($ 30/run). Under these conditions, and on the basis of the labor versus computer expense only, the digital technique was judged to be the more economical. Likewise the analog model component cost, normally not involved in the digital model, represents considerable computer time and further made the digital method more economically attractive in this case.

A distinctive feature of the electric analog simulator is that the variables of the system under study are represented by analogous physical quantities and pieces of equipment. For instance, the electrical resistor and capacitor network will bear a close resemblance in form to the scaled-down version of the aquifer. The pulse generator can be thought of as a large pump connected to a distribution system which controls all individual water extraction rates. And, the oscilloscope is analogous to a water-level recorder. Many other analogies can be drawn that sometimes are of value in gaining a closer understanding of the physical problem under study as compared to the artificial computational steps involved in the digital computer solution of the same problem.

Turning to the category where comparative models were not constructed the following comments seem appropriate. The analog technique offers advantages for studying aquifer response under conditions requiring large numbers of nodes for their solution. In addition, the analog method, being a continuous-time simulator has an advantage over the digital computer in the case of rapidly changing heads, particularly when nonlinear boundaries are included. The advantage of the analog technique in the above categories is because the digital approach requires both space and time variable discretizing; the first of which is limited, on a practical basis by the size of the available computer core storage and the second of which makes accurate solution costly because of the small time increments involved.

The digital computer has advantages in studying aquifer response under the nonlinear conditions of varying transmissivity in unconfined aquifers and in simulating salt water intrusion and other multiphase phenomena (Bittinger, 1967). Comparative models are not necessary here because, in most cases, the mentioned nonlinearities go beyond the reasonable type of iterative or approximate solutions that must accompany the electric analog techniques. This is not to say, however, that one would chose the digital computer to effect a solution in all cases dealing with these particular nonlinearities. For instance, Jacob (1944) gave a method that can be used for approximating the response to pumping of an unconfined aquifer by mathematically adjusting data taken from a model designed under confined conditions. The method produces acceptable results in electric analog simulation under aquifer dewatering conditions up to 25 percent of the total thickness of the aquifer.

CONCLUSIONS

On the basis of the aquifer problems studied it is concluded that the digital method does not require as much time for model construction and data readout and is more convenient than the equipment manipulation phases needed in the analog technique. Also, the digital technique offered more versitality than the analog technique in solving nonlinear boundary problems. However, the electric analog techniques have their advantages for very large problems that would require many time increments and large core storage for an accurate and practical solution. The analog technique offers the advantage when it is desired to solve problems requiring a direct insight into the behavior of the physical system. Factors related to accessibility of equipment and costs dictate to a large extent the simulator technique choice.

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